Alexander von Humboldt
Stiftung/Foundation

Spacer losowy z rzadkimi losowymi zaburzeniami

Piotr Dyszewski (TU München)

27 kwietnia 2021

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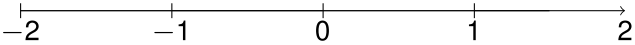

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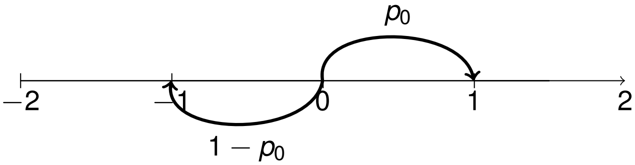

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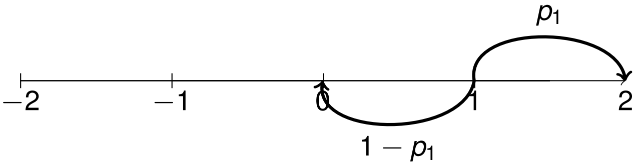

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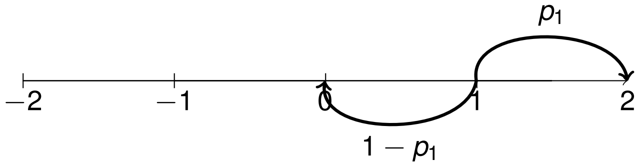

$X = \{X_n\}_{n \in \mathbb{N}}$ spacer losowy w losowym środowisku

$$\mathrm{P}_\omega[X_{n+1} = k+1 \,|X_n = k] = p_k$$
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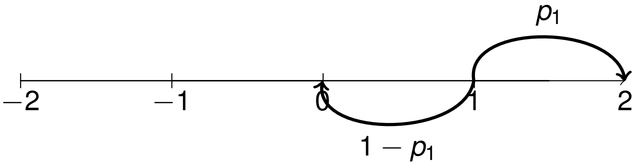

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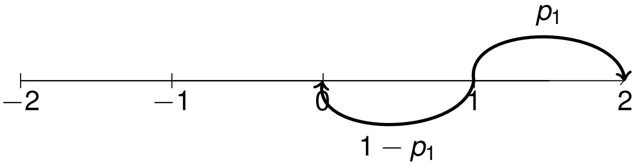

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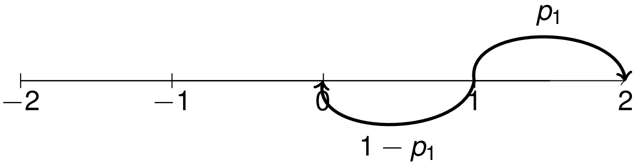

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$$X_n \sim ? \qquad n \to \infty$$

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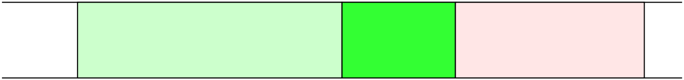
- $\mathbb{E}\sigma^\alpha = 1$ dla $\alpha \in (0,1)$ implikuje
  $$\frac{X_n}{n^\alpha} \to^d \mathcal{L}_\alpha^{-\alpha}$$
  $\mathcal{L}_\alpha$-α stabilna

Rzadkie losowe zaburzenia

[Matzavinos, Roitershtein and Seol, *Electron. J. Probab.* 2017]



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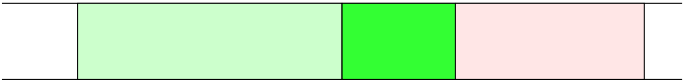

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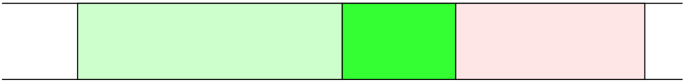

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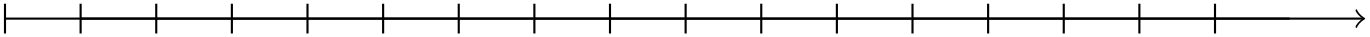

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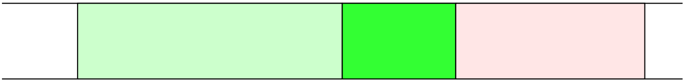

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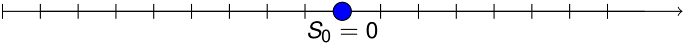

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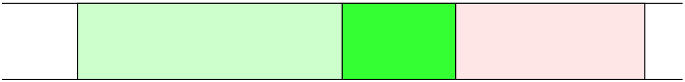

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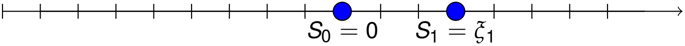

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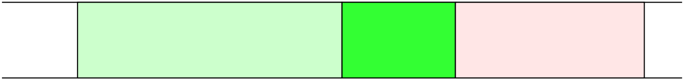

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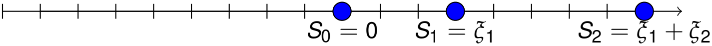

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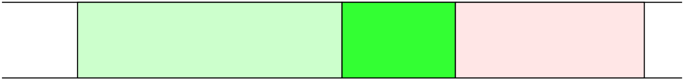

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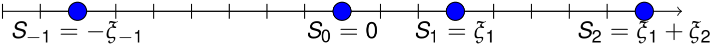

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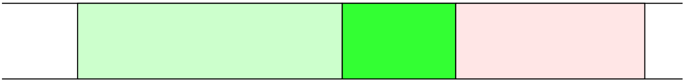

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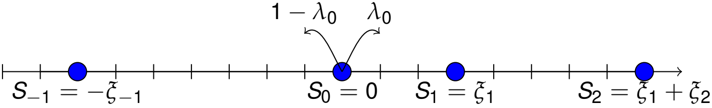

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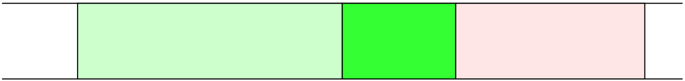

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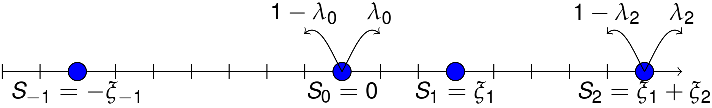

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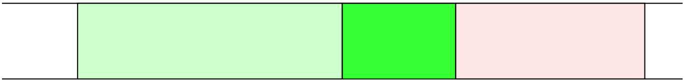

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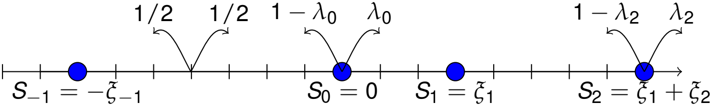

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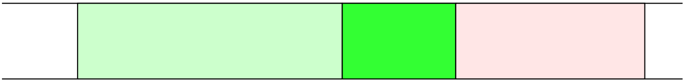

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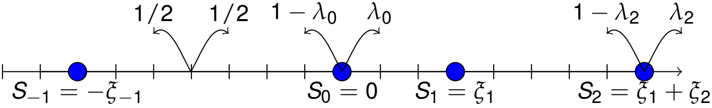
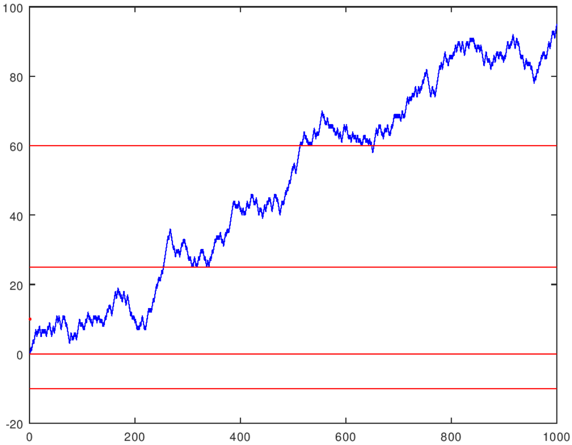


$$p_k = \begin{cases} \lambda_n, & k = S_n \text{ dla pewnego } n \in \mathbb{Z} \\ \frac{1}{2}, & \text{poza tym} \end{cases}$$

$\rho = \frac{1-\lambda}{\lambda}$

Lemat

Jeżeli $\mathbb{E}\log(\rho) < 0$ oraz $\mathbb{E}\log\xi < \infty$, to $X_n \to +\infty$.



Twierdzenie (Buraczewski, D, Iksanov, Marynych, *SPA* 2020)

Niech $\rho = \frac{1-\lambda}{\lambda}$. Załóżmy, że $\mathbb{E}\rho^{\alpha} = 1$ dla pewnej $\alpha \in (0, 1)$

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$$\frac{1}{n^2} T_{S_{V(n)-1}} \to L(\nu)$$

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$$T_n - T_{S_{V(n)-1}} \approx \#\{k \le T_n : X_k \in (S_{V(n)-1}, n)\}$$

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$X' = (X'_n)_n$ prosty spacer losowy odbity w 0.

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$$\frac{1}{\sqrt{n}} X'_{[nt]} \Rightarrow |B_t|$$

B - ruch Browna

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B - ruch Browna

$$\left(n^{-2} T'(nt)\right)_{t \ge 0} \Rightarrow (M(t))_{t \ge 0}, \qquad M(t) = \inf\{s > 0 \,:\, |B_s| = t\}$$

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B - ruch Browna

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$$\frac{S_{V(n)-1}}{n} \to \zeta_\nu \in (0,1)$$

$$n^{-2}(T_n - T_{S_{V(n)-1}}) \to M(1 - \zeta_\nu)$$

$\mathbb{P}(\xi > x) \sim x^{-\beta}$, $\beta \in (0,1)$, $\beta/2 < \alpha$

$$V(n) = \inf\{k \,:\, S_k > n\}$$

$$n^{-\beta} V(n) \to \nu$$

$$\frac{S_{V(n)-1}}{n} \to \zeta_\nu \in (0,1)$$

$L(t) -$ Lévy subordinator, $\qquad M(t) = \inf\{s > 0 \,:\, |B_s| = t\}$

$$\frac{1}{n^2} T_n \to^d L(\nu) + M(1 - \zeta_\nu)$$

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$$\frac{1}{n^2} T_n \to^d L(\nu) + M(1-\zeta_\nu) = L(\nu) + M(1)(1-\zeta_\nu)^2$$

$$\frac{1}{\sqrt{n}} X_n \to \chi = (L(\nu) + M(1)(1-\zeta_\nu)^2)^{1/2}$$

D. Buraczewski, P. Dyszewski, A. Iksanov and A. Marynych.
Random walks in a strongly sparse random environment.
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Random walks in a random environment.
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Walks and trees are abstractly identical objects
(T. E. Harris ’52)

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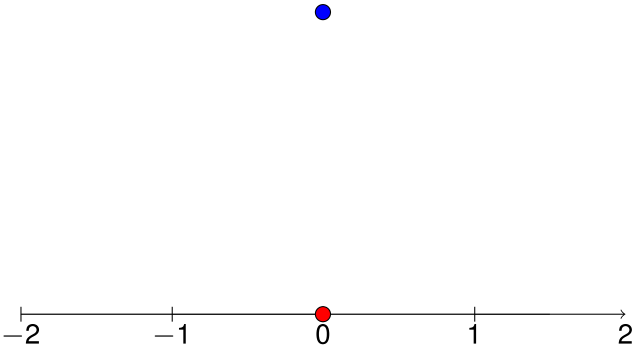
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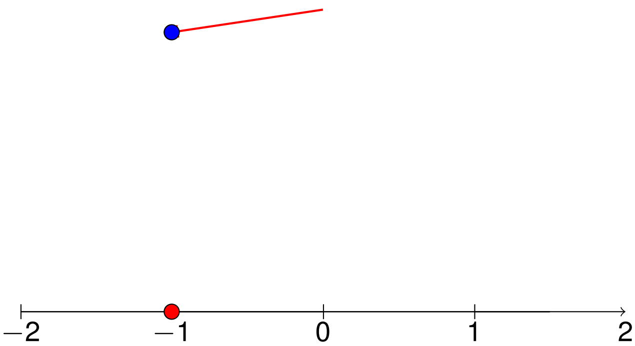
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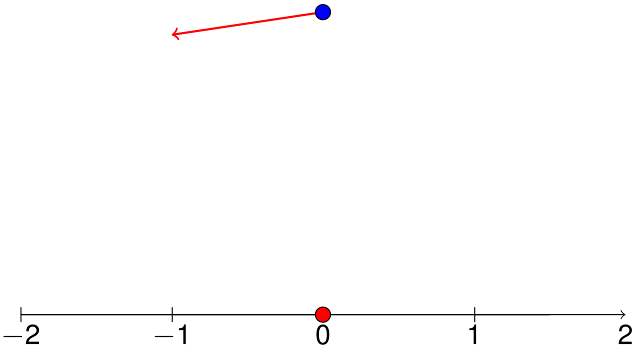
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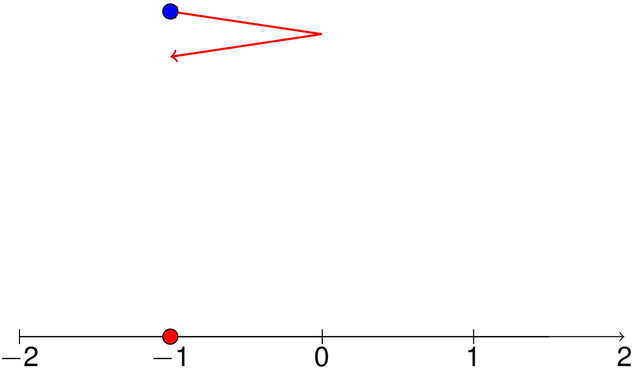

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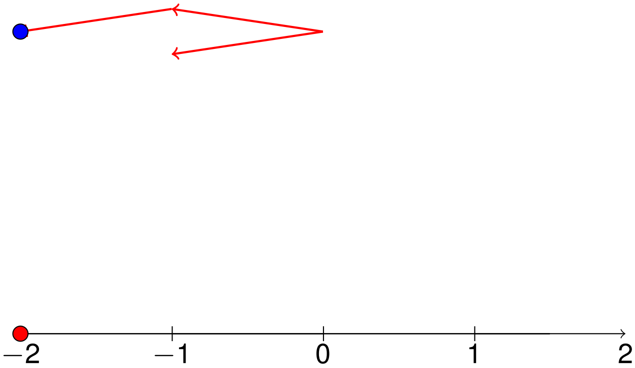
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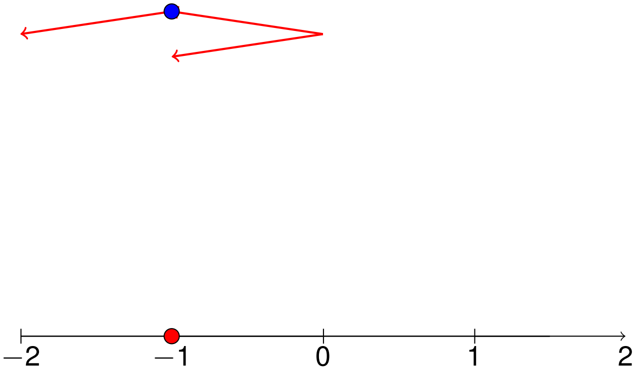

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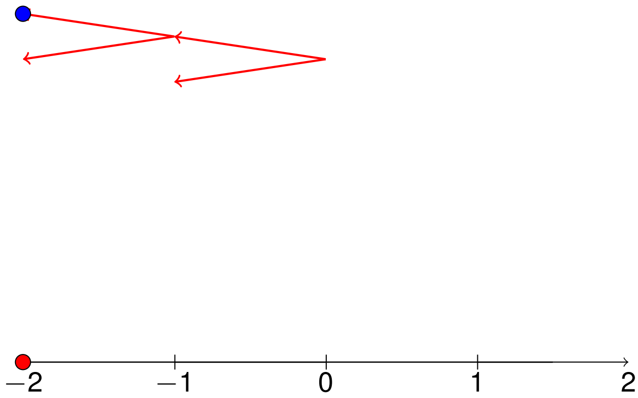
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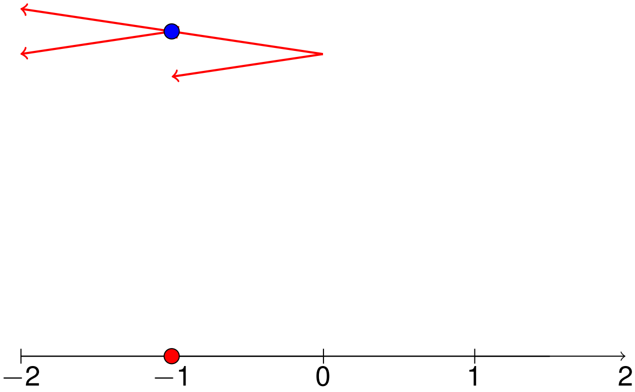
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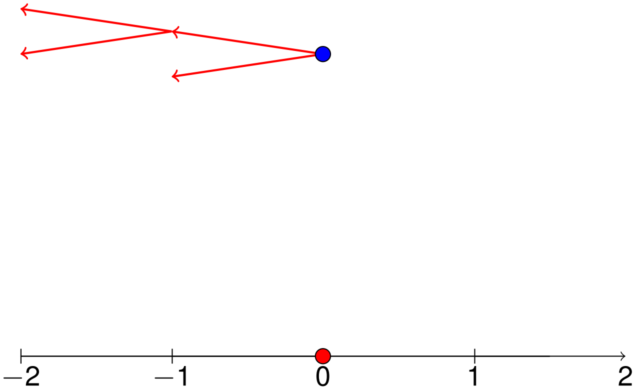
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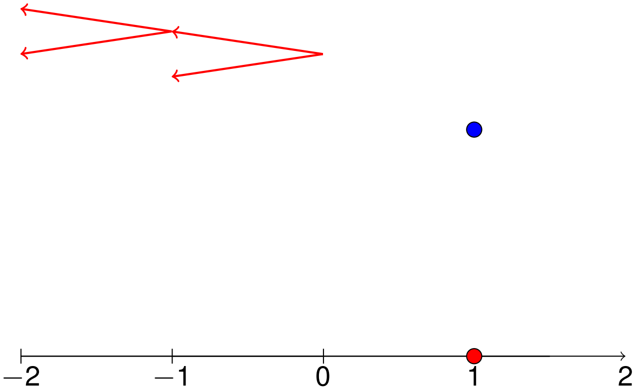
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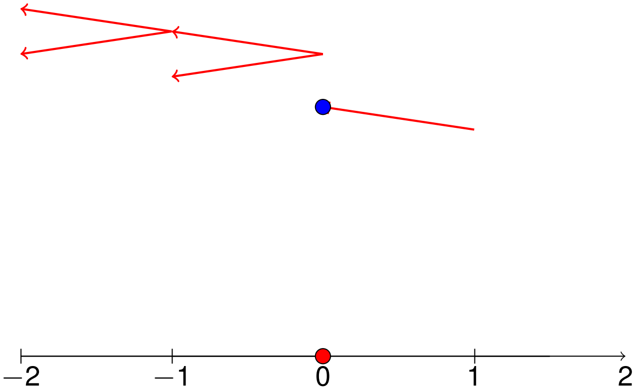

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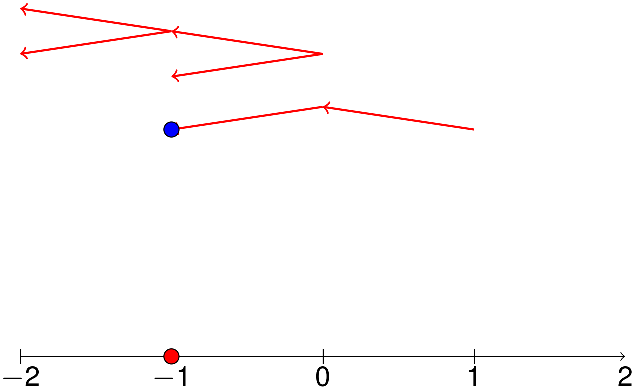
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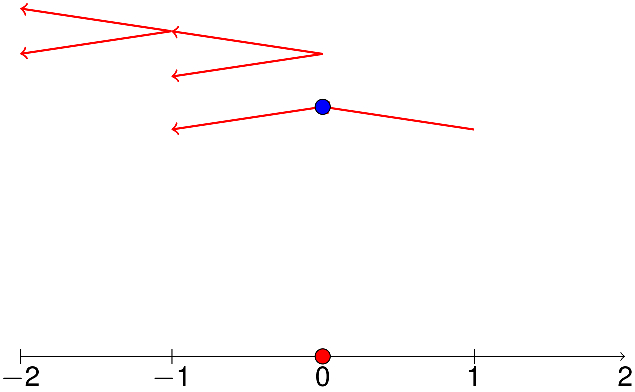

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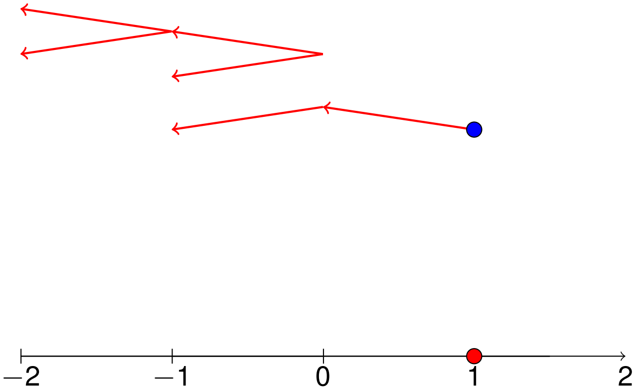
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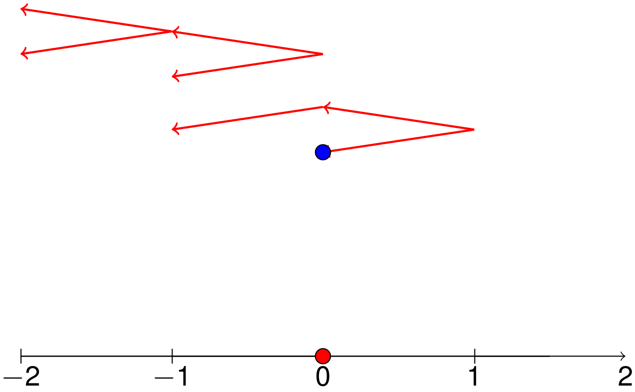

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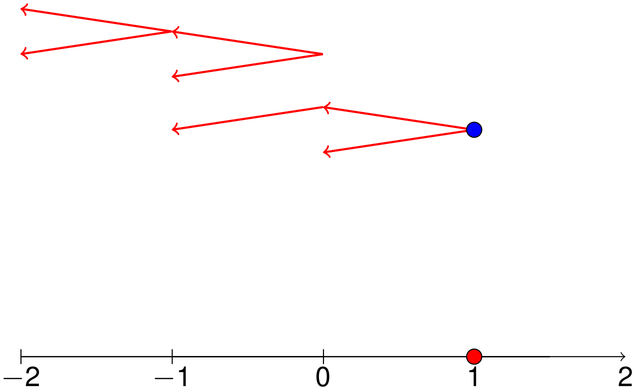
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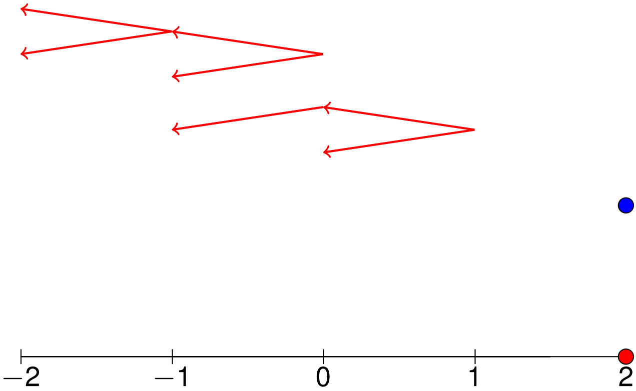
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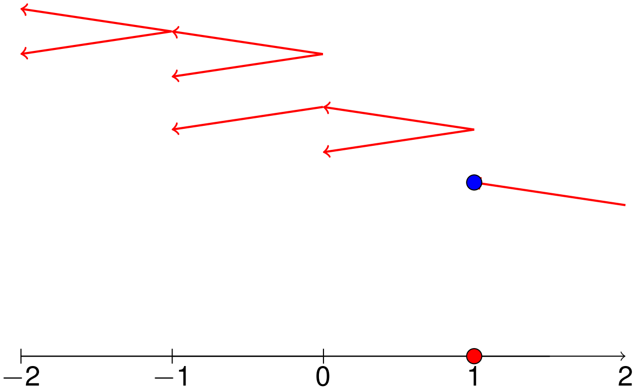

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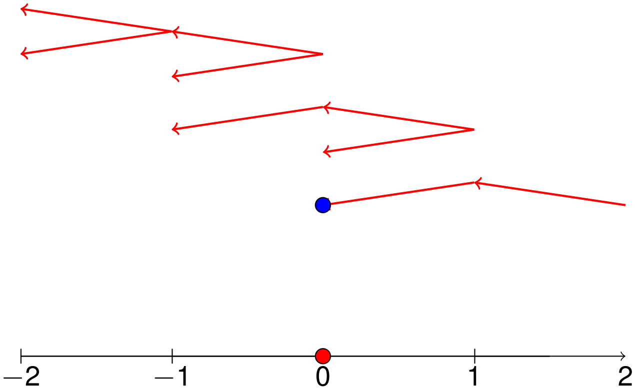

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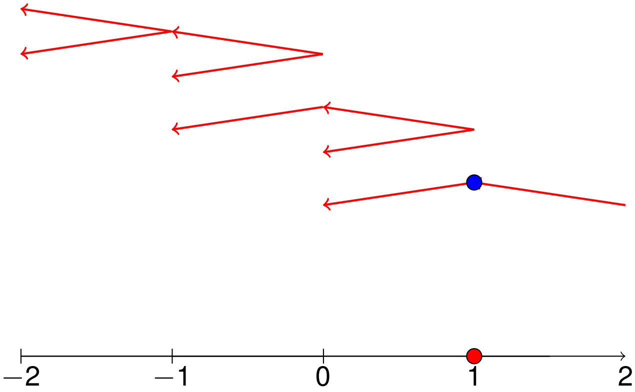

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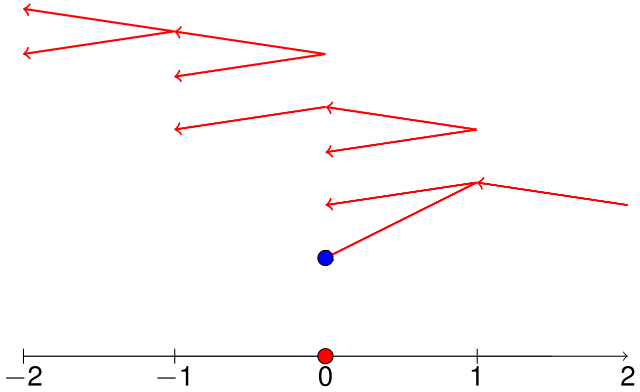

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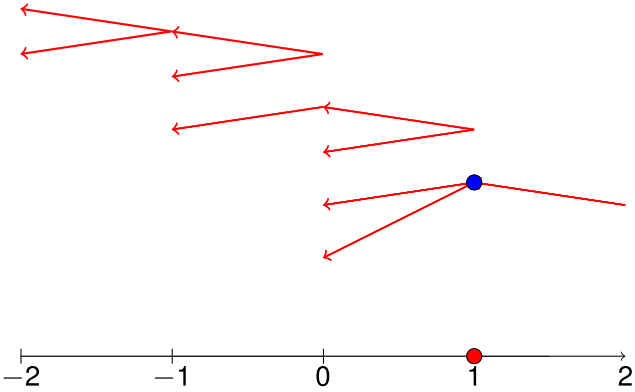

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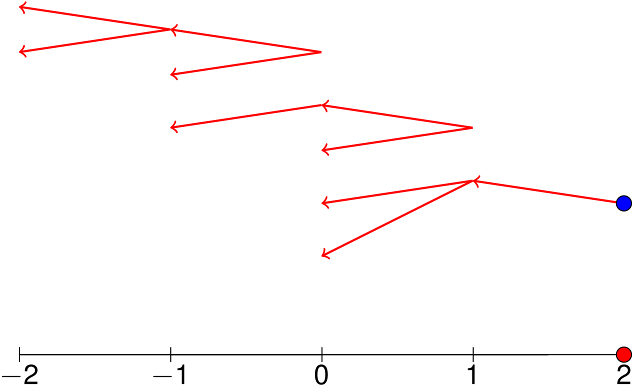
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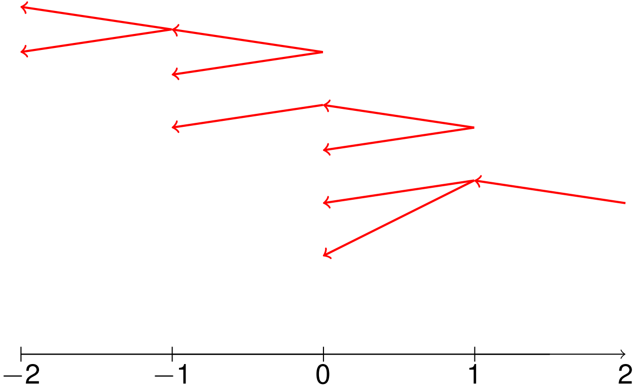

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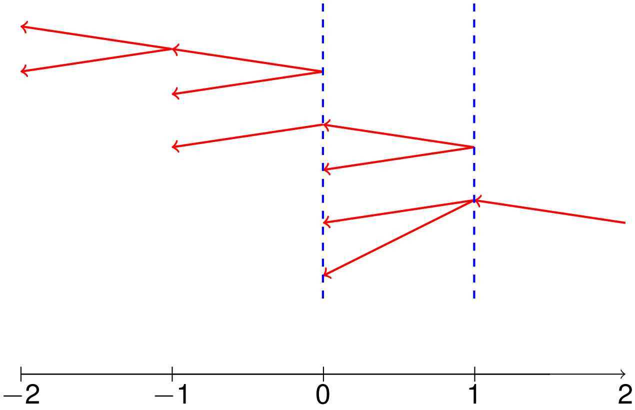
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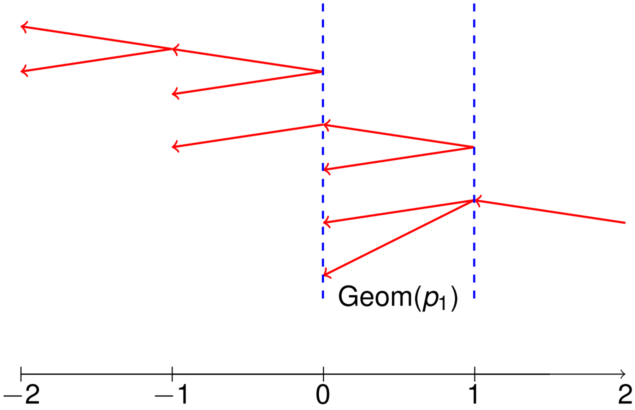
Geom(p_1)
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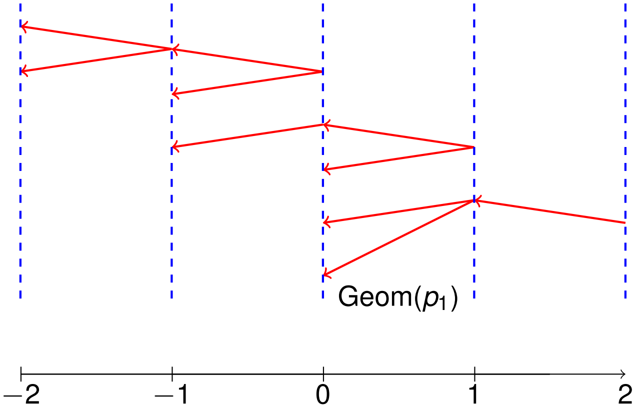

Geom(p_1)
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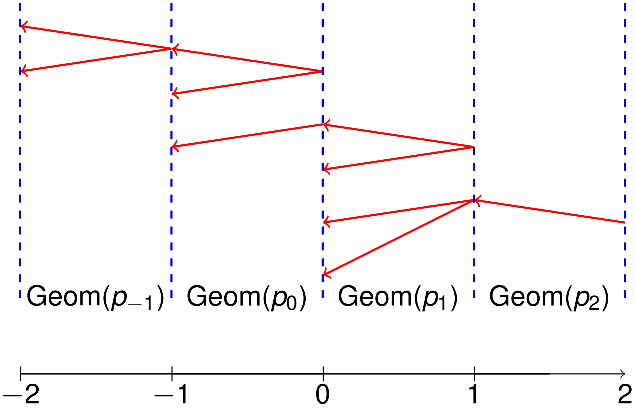

Geom(p_{-1})
Geom(p_0)
Geom(p_1)
Geom(p_2)
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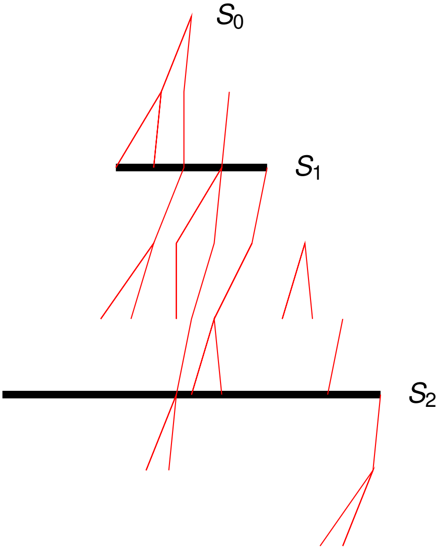
S_0
S_1
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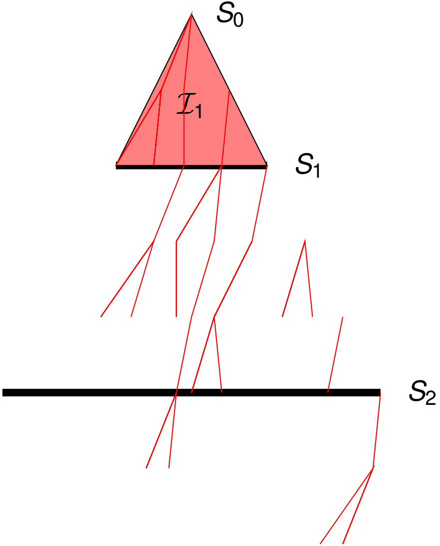
S_0
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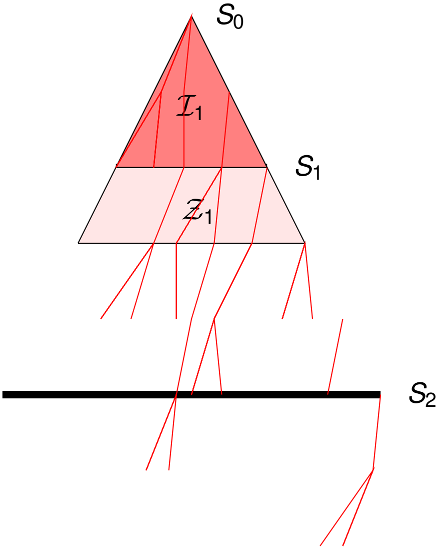
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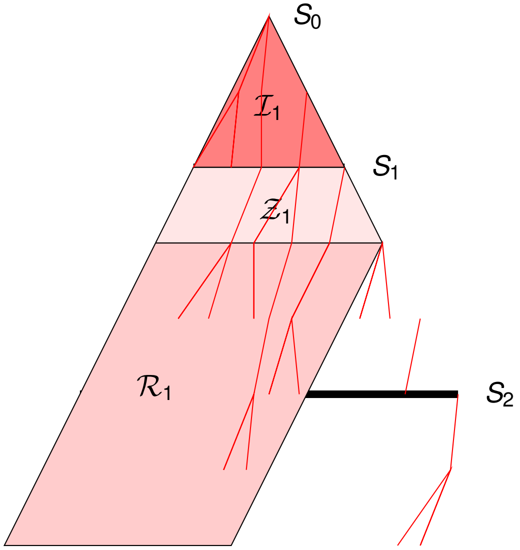
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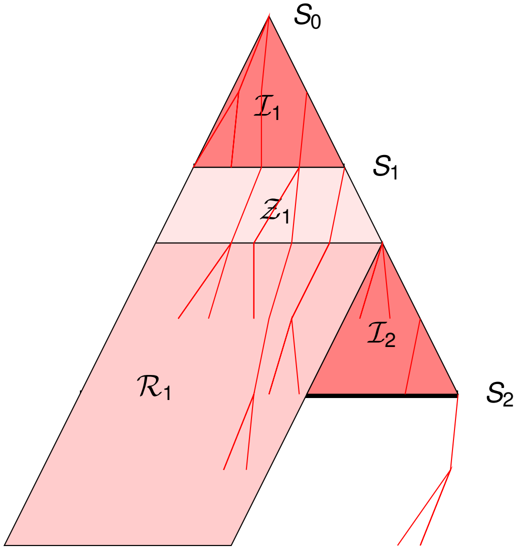
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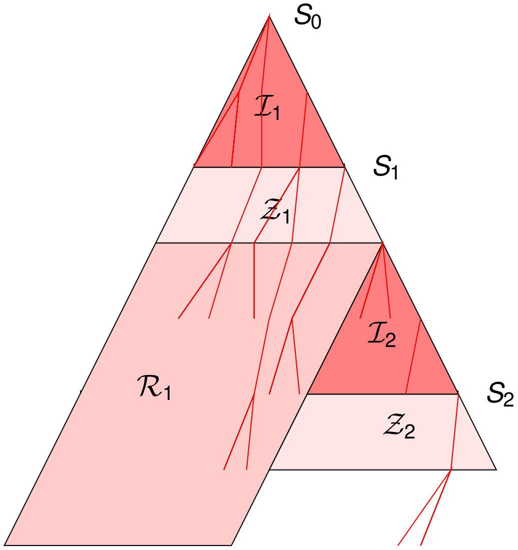

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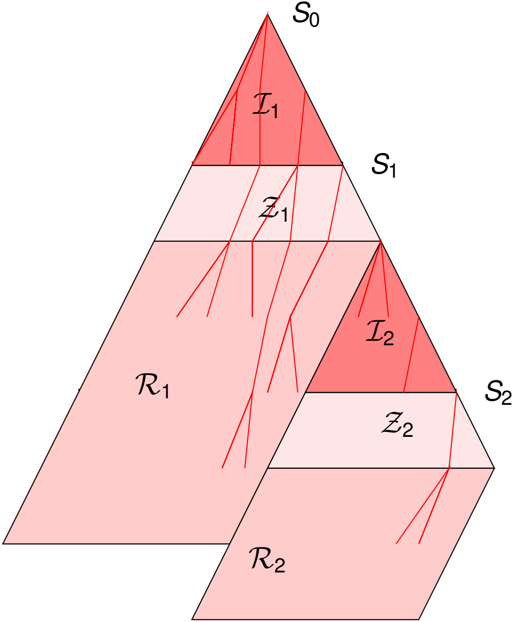
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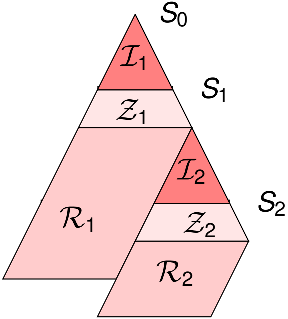
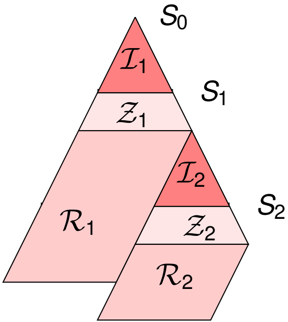


$$T_{S_n} \stackrel{d}{=} S_n + 2\sum_{k=1}^{n} \mathcal{I}_k + \mathcal{Z}_k + \mathcal{R}_k$$



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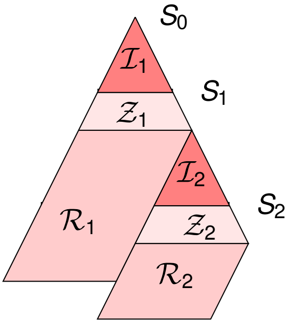
$$\mathrm{E}_\omega \mathcal{I}_k \approx \xi_k^2$$



$$T_{S_n} \stackrel{d}{=} S_n + 2\sum_{k=1}^{n} \mathcal{I}_k + \mathcal{Z}_k + \mathcal{R}_k$$

$$\mathrm{E}_\omega \mathcal{I}_k \approx \xi_k^2$$

$$\mathbb{P}[\mathcal{I}_k + \mathcal{Z}_k + \mathcal{R}_k > t] \sim t^{-\gamma} \quad \gamma = \alpha \wedge \frac{\beta}{2}$$