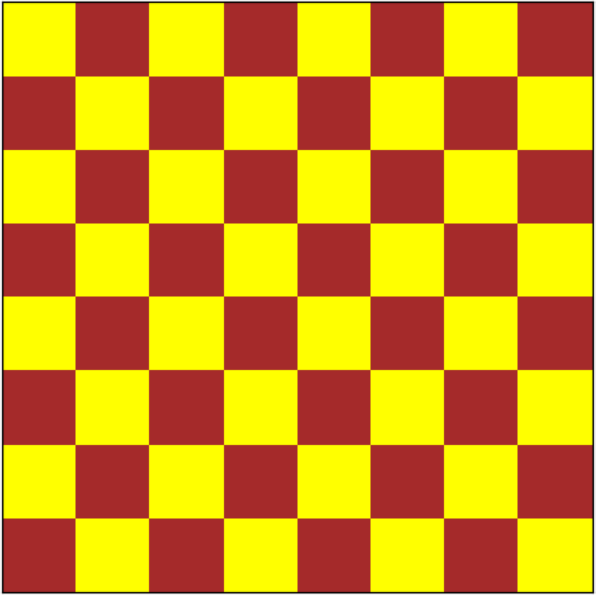
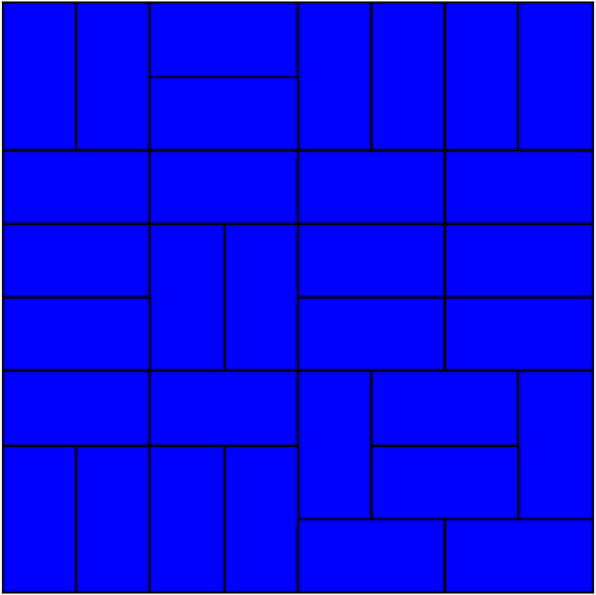
Domino, skarby Azteków i koło podbiegunowe

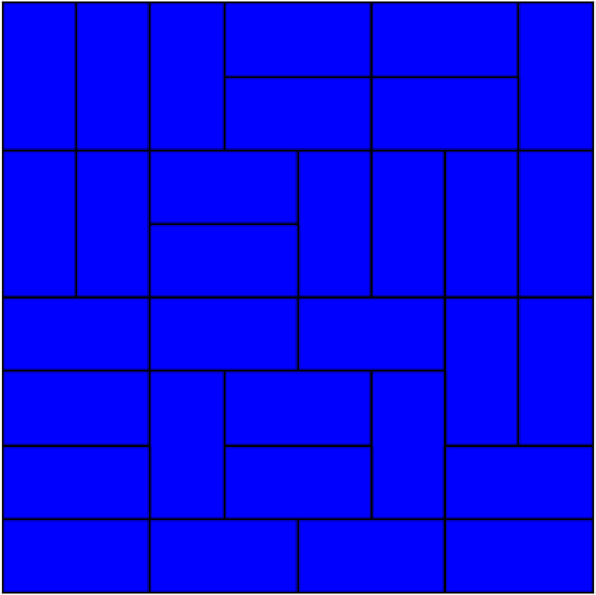
Piotr Dyszewski

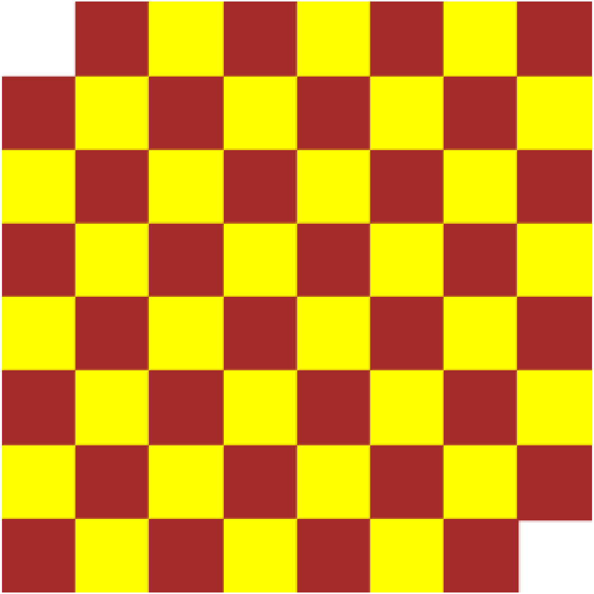
Uniwersytet Wrocławski

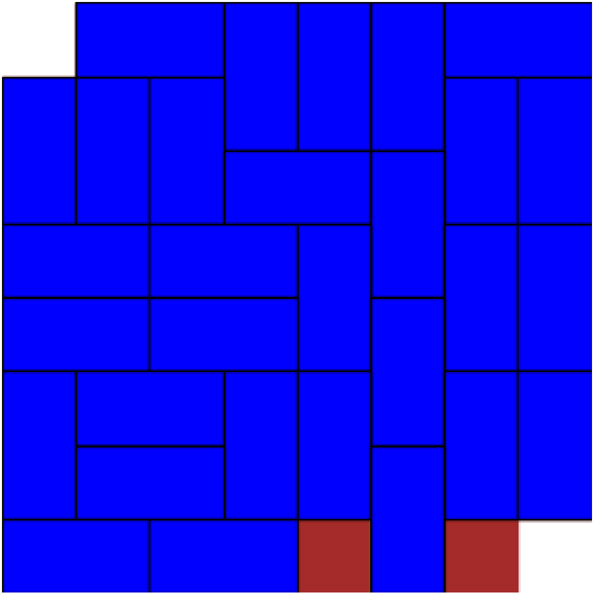
10 września 2024

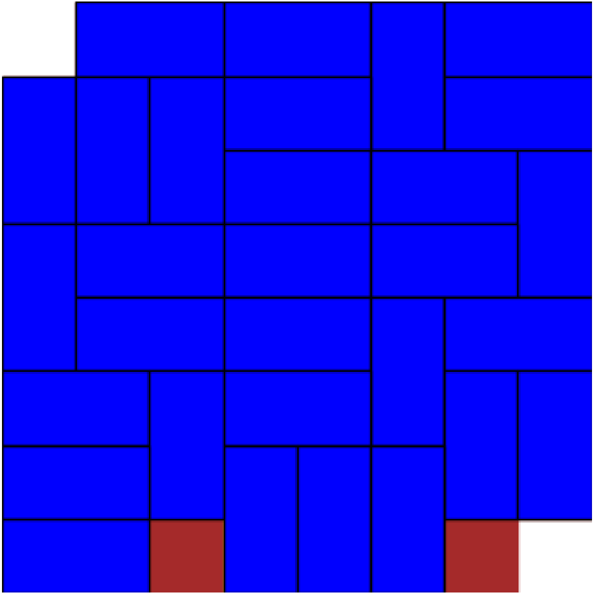


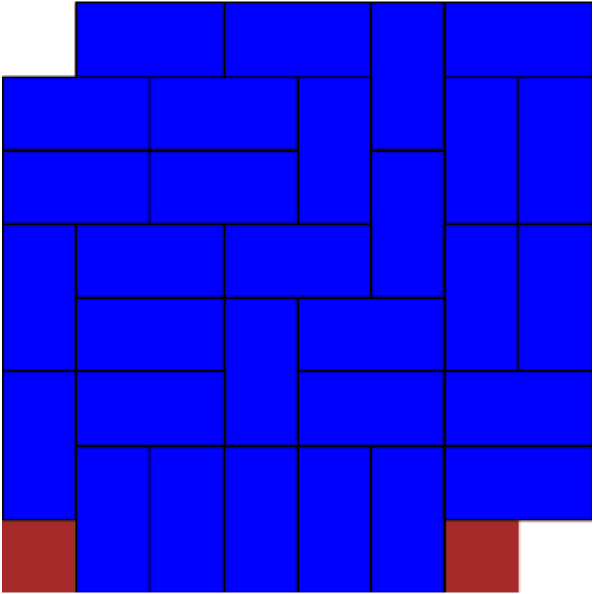


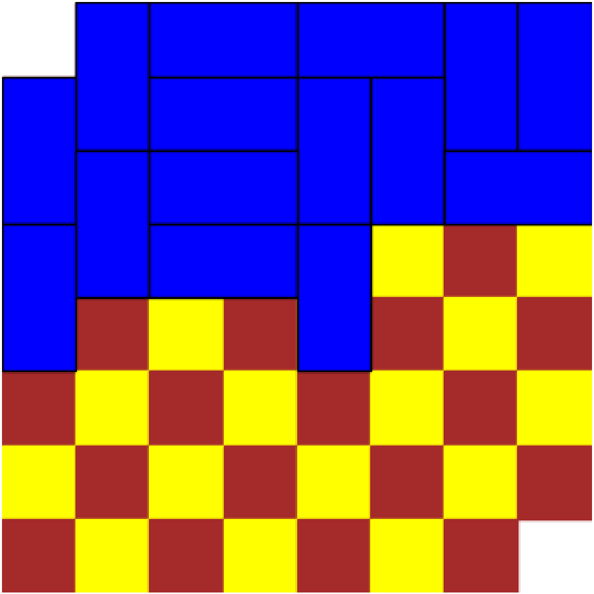


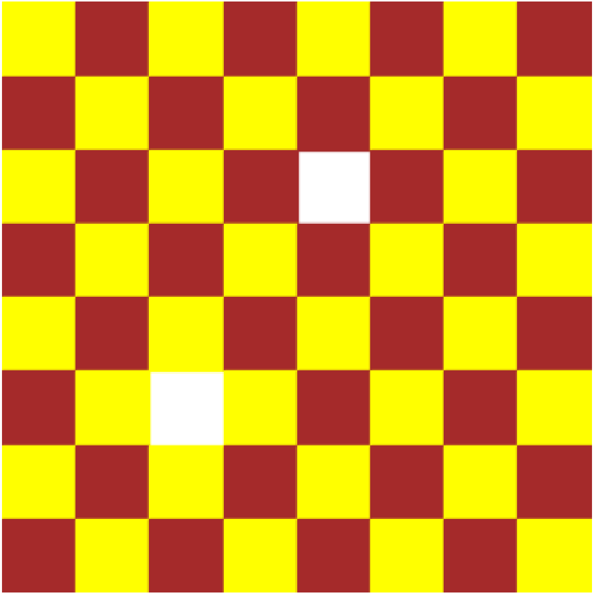


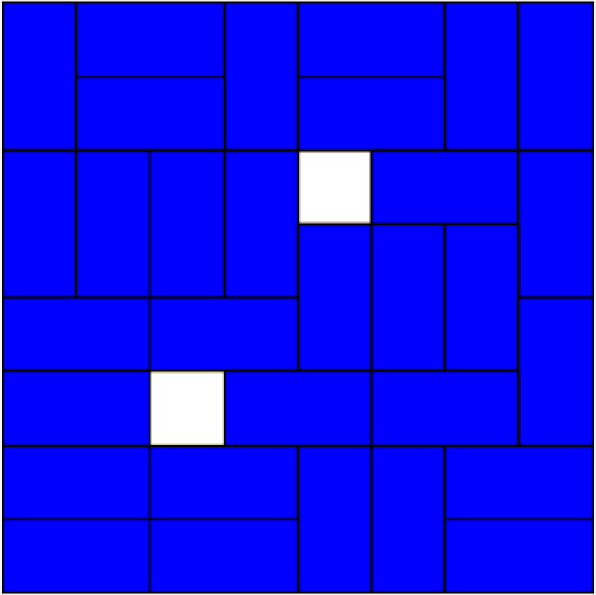


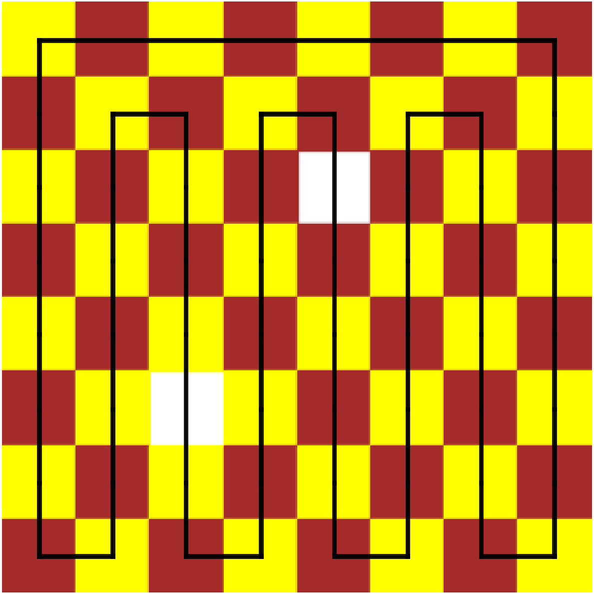






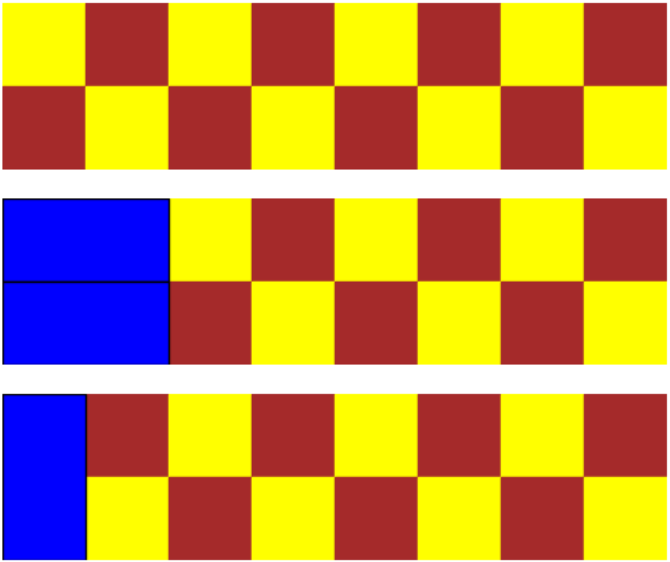






$H_n =$ liczba parkietaży szachownicy 2xn

$H_n =$ liczba parkietaży szachownicy 2xn



H_n = liczba parkietaży szachownicy 2xn

$H_n = H_{n-1} + H_{n-2}, \quad n \geq 3$

$H_1 = 1$, $H_2 = 2$.

H_1	H_2	H_3	H_4	H_5	H_6	H_7	H_8	H_9	H_{10}
1	2	3	5	8	13	21	34	55	89

$$F_n = F_{n-1} + F_{n-2}, \quad n \geq 3$$
$$F_1 = 1,\ F_2 = 1.$$

F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8	F_9	F_{10}	F_{11}
1	1	2	3	5	8	13	21	34	55	89

$F_n = F_{n-1} + F_{n-2}, \quad n \geq 3$
$F_1 = 1,\ F_2 = 1.$

F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8	F_9	F_{10}	F_{11}
1	1	2	3	5	8	13	21	34	55	89

$$F_n = \frac{\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n}{\sqrt{5}}$$

H_n = liczba parkietaży szachownicy 2xn

$H_n = H_{n-1} + H_{n-2}, \quad n \geq 3$
$H_1 = 1$, $H_2 = 2$.

H_1	H_2	H_3	H_4	H_5	H_6	H_7	H_8	H_9
1	2	3	5	8	13	21	34	55

$$H_n = F_{n+1} = \frac{\left(\frac{1+\sqrt{5}}{2}\right)^{n+1} - \left(\frac{1-\sqrt{5}}{2}\right)^{n+1}}{\sqrt{5}}$$

$B_{n,m}$ = liczba parkietaży szachownicy *nxm*

Kasteleyn, P. W. 1961

Physica 27 1209–1225

THE STATISTICS OF DIMERS ON A LATTICE

I. THE NUMBER OF DIMER ARRANGEMENTS ON A QUADRATIC LATTICE

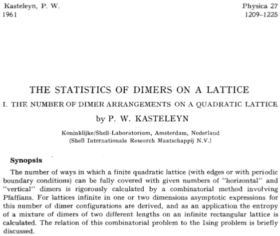
by P. W. KASTELEYN

Koninklijke/Shell-Laboratorium, Amsterdam, Nederland
(Shell Internationale Research Maatschappij N.V.)

Synopsis

The number of ways in which a finite quadratic lattice (with edges or with periodic boundary conditions) can be fully covered with given numbers of "horizontal" and "vertical" dimers is rigorously calculated by a combinatorial method involving Pfaffians. For lattices infinite in one or two dimensions asymptotic expressions for this number of dimer configurations are derived, and as an application the entropy of a mixture of dimers of two different lengths on an infinite rectangular lattice is calculated. The relation of this combinatorial problem to the Ising problem is briefly discussed.

$B_{n,m}$ = liczba parkietaży szachownicy *nxm*


Kasteleyn, P. W. 1961 — Physica 27 1209–1225

THE STATISTICS OF DIMERS ON A LATTICE

I. THE NUMBER OF DIMER ARRANGEMENTS ON A QUADRATIC LATTICE

by P. W. KASTELEYN

Koninklijke/Shell-Laboratorium, Amsterdam, Nederland
(Shell Internationale Research Maatschappij N.V.)

Synopsis

The number of ways in which a finite quadratic lattice (with edges or with periodic boundary conditions) can be fully covered with given numbers of "horizontal" and "vertical" dimers is rigorously calculated by a combinatorial method involving Pfaffians. For lattices infinite in one or two dimensions asymptotic expressions for this number of dimer configurations are derived, and as an application the entropy of a mixture of dimers of two different lengths on an infinite rectangular lattice is calculated. The relation of this combinatorial problem to the Ising problem is briefly discussed.

$$B_{n,m} = \prod_{j=1}^{\lceil \frac{m}{2} \rceil} \prod_{k=1}^{\lceil \frac{n}{2} \rceil} \left(4\cos^2\left(\frac{j\pi}{m+1}\right) + 4\cos^2\left(\frac{k\pi}{n+1}\right) \right)$$

$B_{n,m}$ = liczba parkietaży szachownicy nxm

$$B_{n,m} = \prod_{j=1}^{\lceil \frac{m}{2} \rceil} \prod_{k=1}^{\lceil \frac{n}{2} \rceil} \left(4\cos^2\left(\frac{j\pi}{m+1}\right) + 4\cos^2\left(\frac{k\pi}{n+1}\right) \right)$$

$B_{8,8}$ = liczba parkietaży szachownicy $8x8$

$$B_{8,8} = \prod_{j=1}^{4} \prod_{k=1}^{4} \left(4\cos^2\left(\frac{j\pi}{9}\right) + 4\cos^2\left(\frac{k\pi}{9}\right) \right)$$

$B_{8,8}$ = liczba parkietaży szachownicy $8x8$

$$B_{8,8} = \prod_{j=1}^{4}\prod_{k=1}^{4}\left(4\cos^2\left(\frac{j\pi}{9}\right) + 4\cos^2\left(\frac{k\pi}{9}\right)\right) =$$

$$\left(4\cos^2\left(\frac{1\pi}{9}\right) + 4\cos^2\left(\frac{1\pi}{9}\right)\right)\left(4\cos^2\left(\frac{1\pi}{9}\right) + 4\cos^2\left(\frac{2\pi}{9}\right)\right)$$
$$\left(4\cos^2\left(\frac{1\pi}{9}\right) + 4\cos^2\left(\frac{3\pi}{9}\right)\right)\left(4\cos^2\left(\frac{1\pi}{9}\right) + 4\cos^2\left(\frac{4\pi}{9}\right)\right)$$
$$\left(4\cos^2\left(\frac{2\pi}{9}\right) + 4\cos^2\left(\frac{1\pi}{9}\right)\right)\left(4\cos^2\left(\frac{2\pi}{9}\right) + 4\cos^2\left(\frac{2\pi}{9}\right)\right)$$
$$\left(4\cos^2\left(\frac{2\pi}{9}\right) + 4\cos^2\left(\frac{3\pi}{9}\right)\right)\left(4\cos^2\left(\frac{2\pi}{9}\right) + 4\cos^2\left(\frac{4\pi}{9}\right)\right)$$
$$\left(4\cos^2\left(\frac{3\pi}{9}\right) + 4\cos^2\left(\frac{1\pi}{9}\right)\right)\left(4\cos^2\left(\frac{3\pi}{9}\right) + 4\cos^2\left(\frac{2\pi}{9}\right)\right)$$
$$\left(4\cos^2\left(\frac{3\pi}{9}\right) + 4\cos^2\left(\frac{3\pi}{9}\right)\right)\left(4\cos^2\left(\frac{3\pi}{9}\right) + 4\cos^2\left(\frac{4\pi}{9}\right)\right)$$
$$\left(4\cos^2\left(\frac{4\pi}{9}\right) + 4\cos^2\left(\frac{1\pi}{9}\right)\right)\left(4\cos^2\left(\frac{4\pi}{9}\right) + 4\cos^2\left(\frac{2\pi}{9}\right)\right)$$
$$\left(4\cos^2\left(\frac{4\pi}{9}\right) + 4\cos^2\left(\frac{3\pi}{9}\right)\right)\left(4\cos^2\left(\frac{4\pi}{9}\right) + 4\cos^2\left(\frac{4\pi}{9}\right)\right)$$

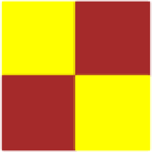
$B_{n,m}$ = liczba parkietaży szachownicy *nxm*

$$B_{n,m} = \prod_{j=1}^{\lceil \frac{m}{2} \rceil} \prod_{k=1}^{\lceil \frac{n}{2} \rceil} \left(4\cos^2\left(\frac{j\pi}{m+1}\right) + 4\cos^2\left(\frac{k\pi}{n+1}\right) \right)$$

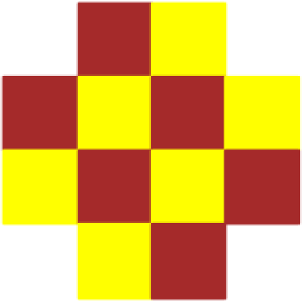
$B_{8,8}$ = liczba parkietaży szachownicy 8*x*8

$$B_{8,8} = \prod_{j=1}^{4} \prod_{k=1}^{4} \left(4\cos^2\left(\frac{j\pi}{9}\right) + 4\cos^2\left(\frac{k\pi}{9}\right) \right)$$
$$= 12\,988\,816$$

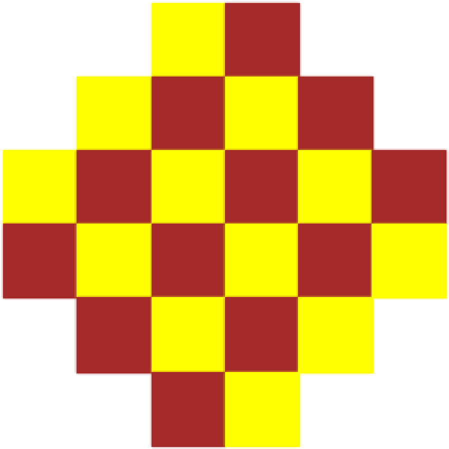
$\mathcal{A}(1)$



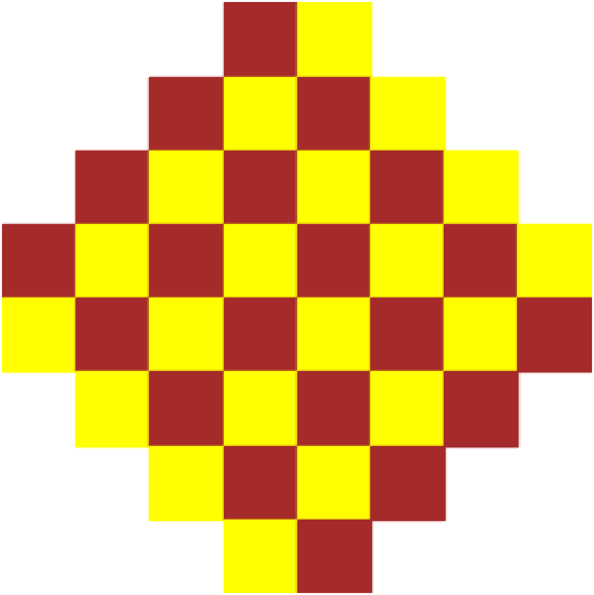
$\mathcal{A}(2)$



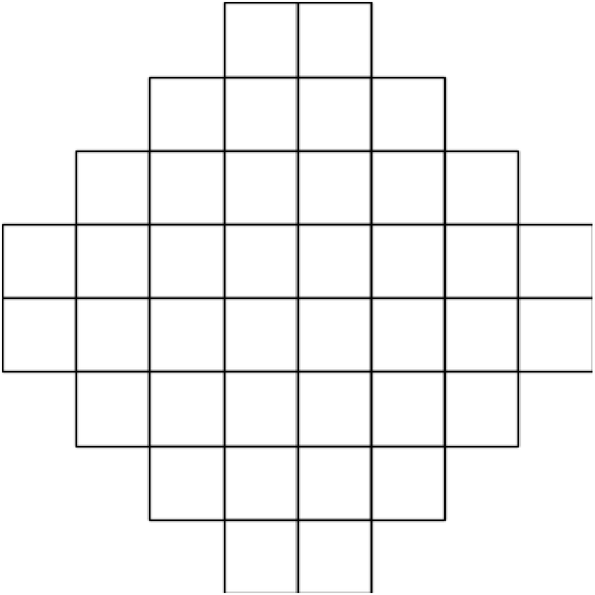
$\mathcal{A}(3)$

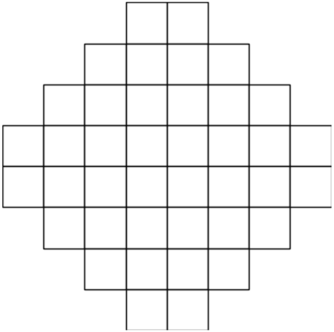


$\mathcal{A}(4)$

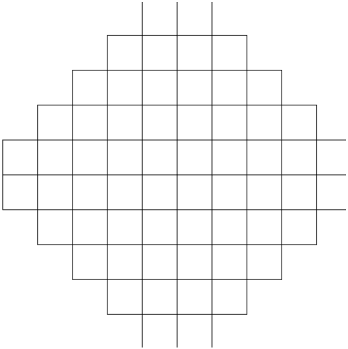


$\mathcal{A}(4)$





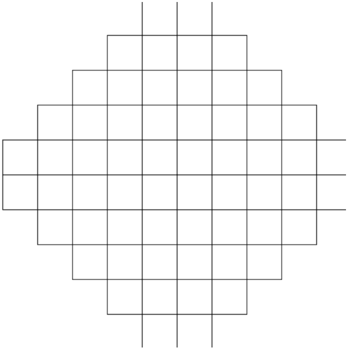
$\mathcal{A}(5)$



$A_n =$ liczba parkietaży $\mathcal{A}(n)$.

$$A_n = 2^{1+2+3+\ldots+n}$$

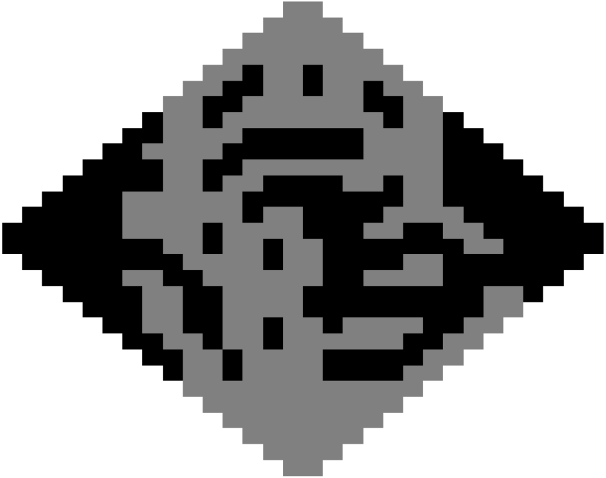
$\mathcal{A}(5)$

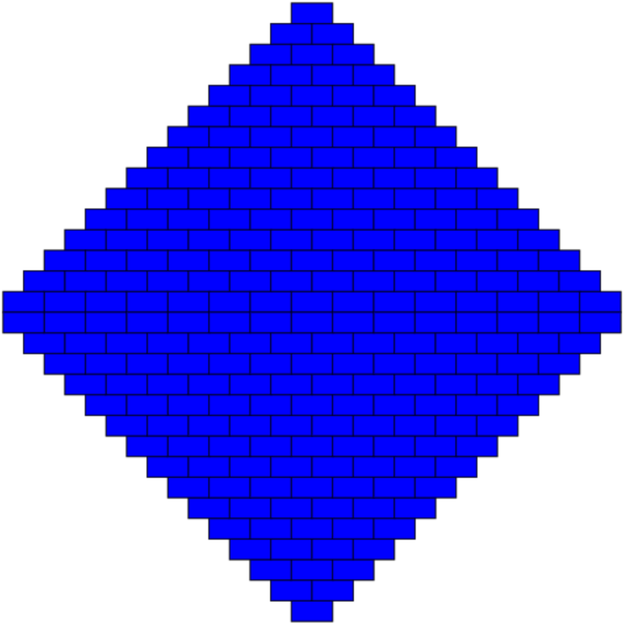


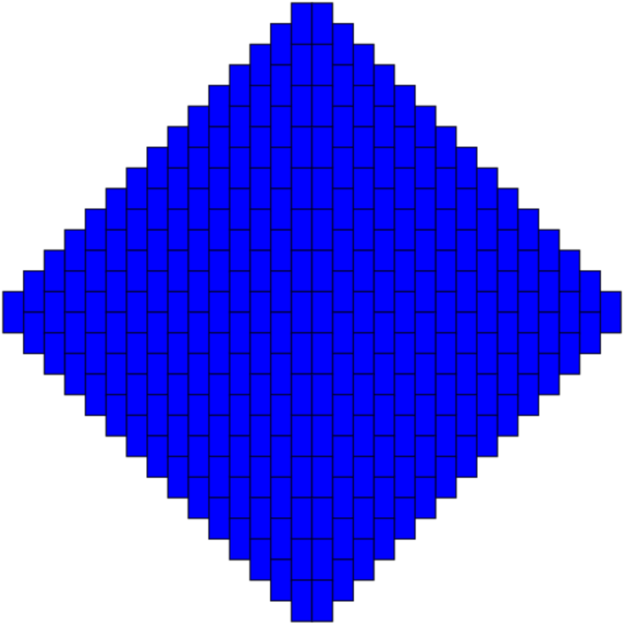
$A_n =$ liczba parkietaży $\mathcal{A}(n)$.

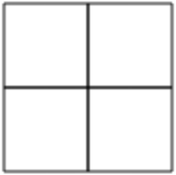
$$A_n = 2^{1+2+3+...+n}$$

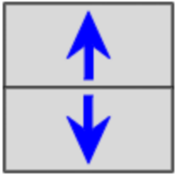
$$A_5 = 2^{15} = 131072$$

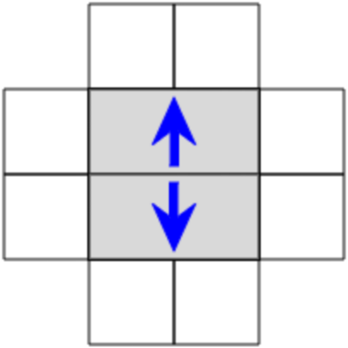


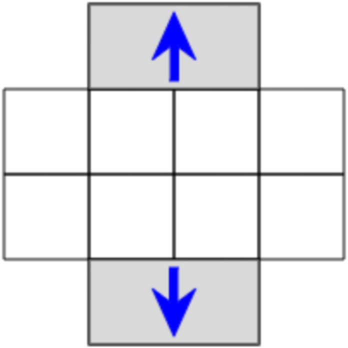


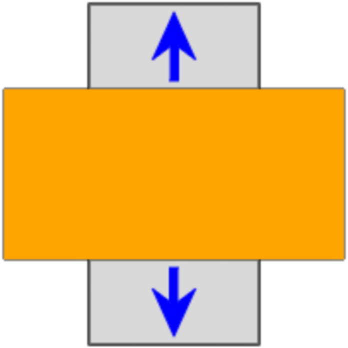


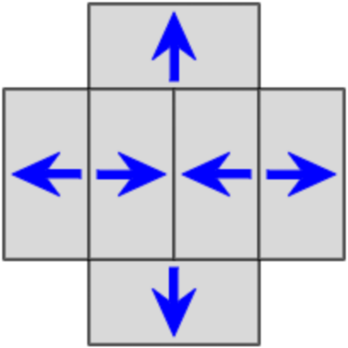


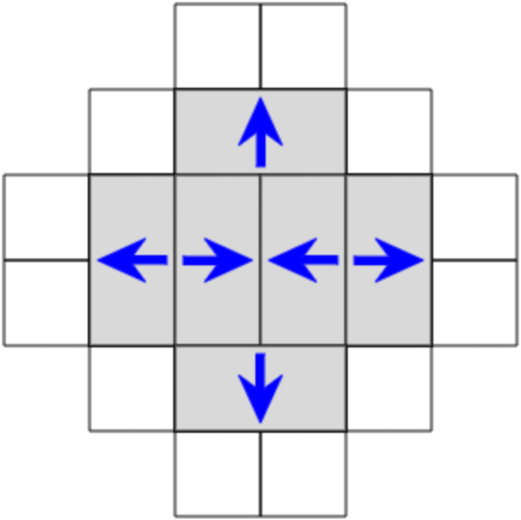


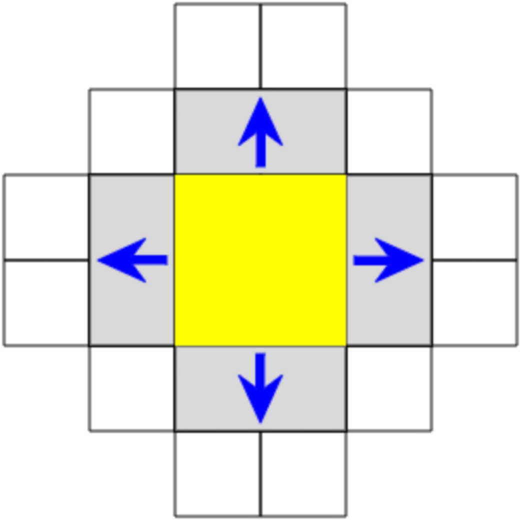


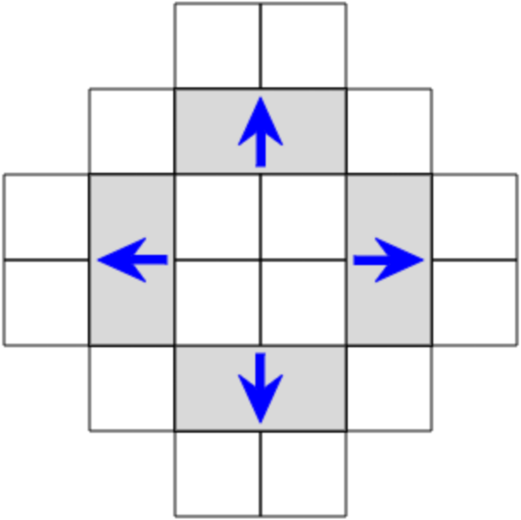


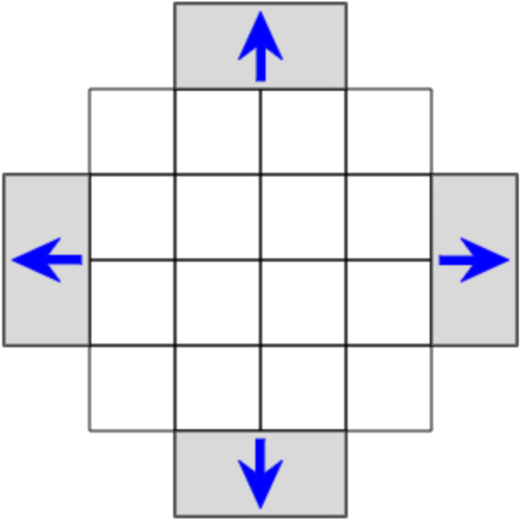


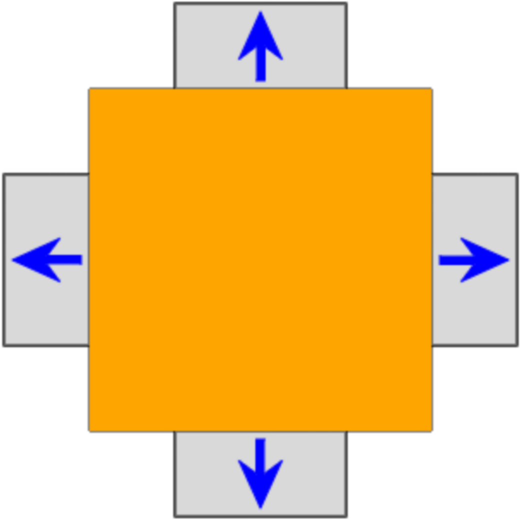


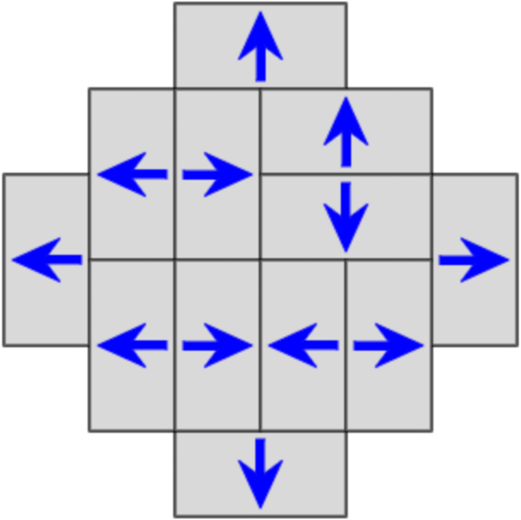


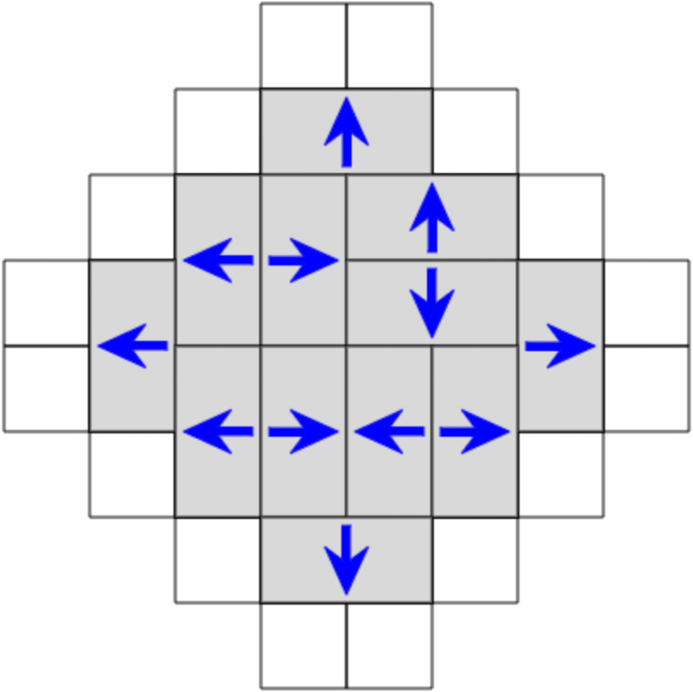


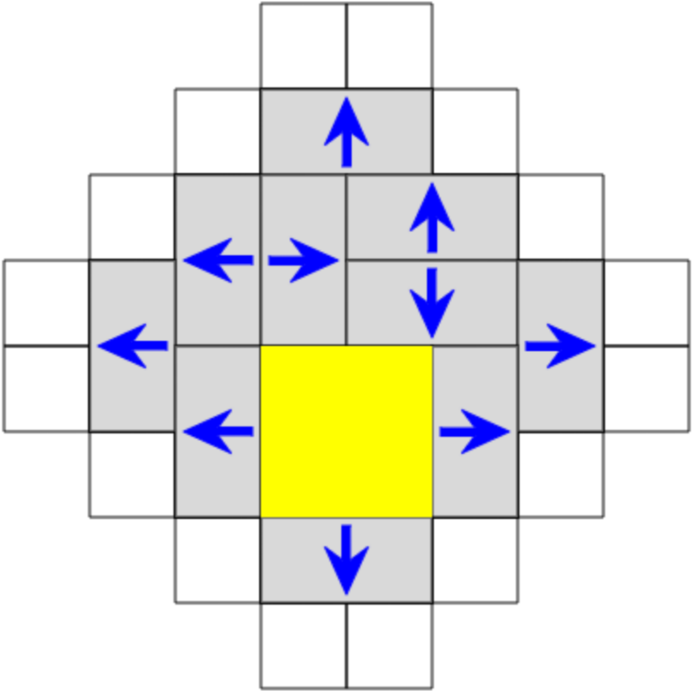


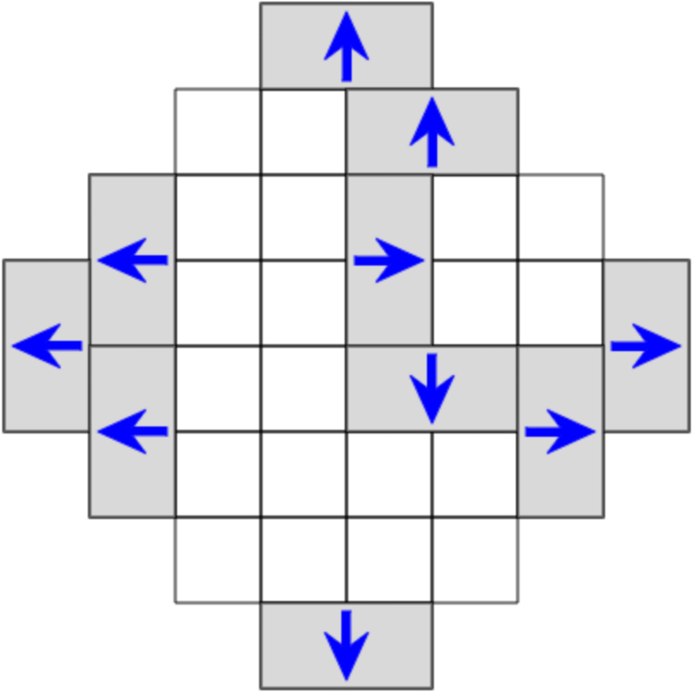


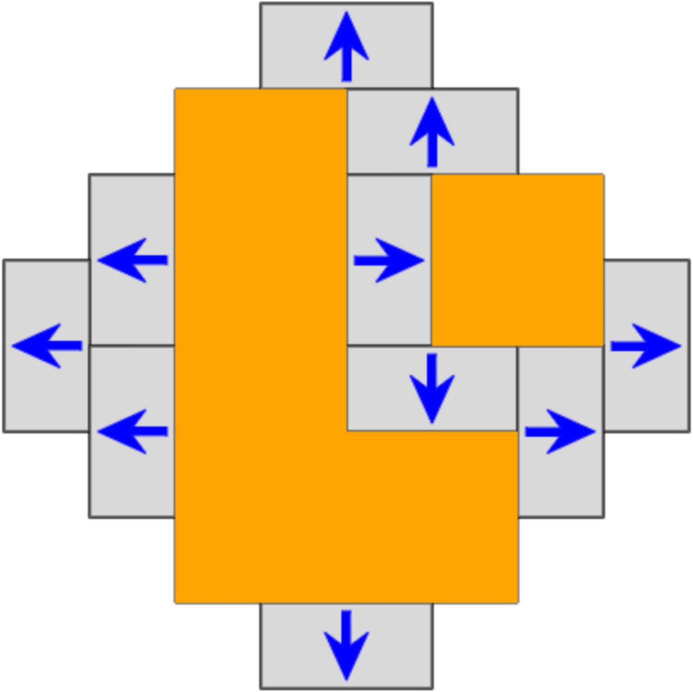


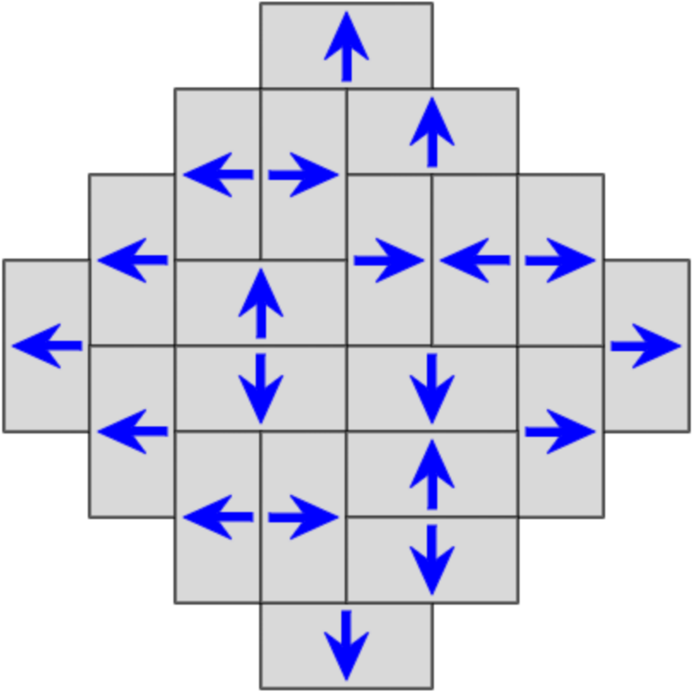


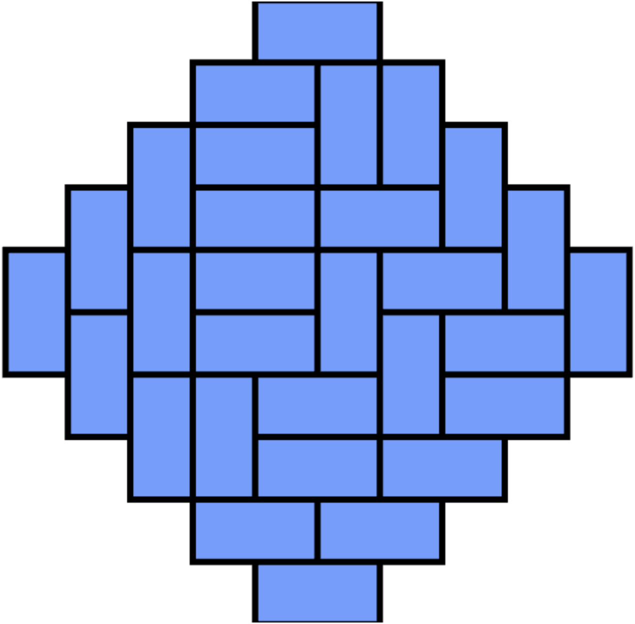


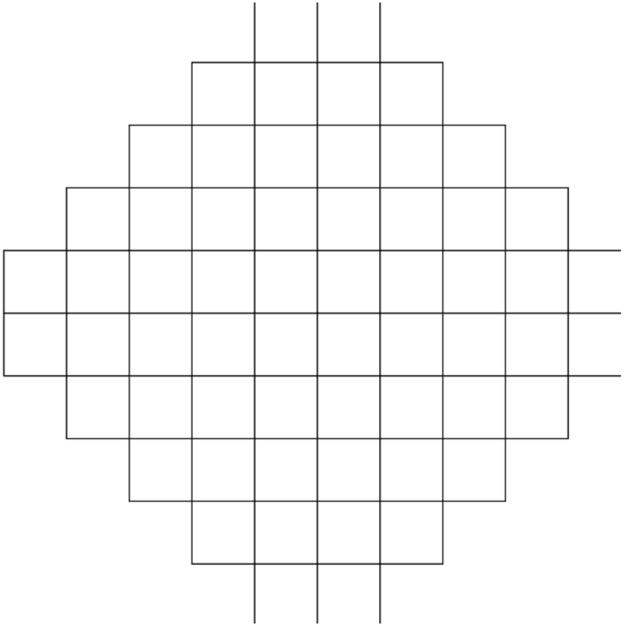


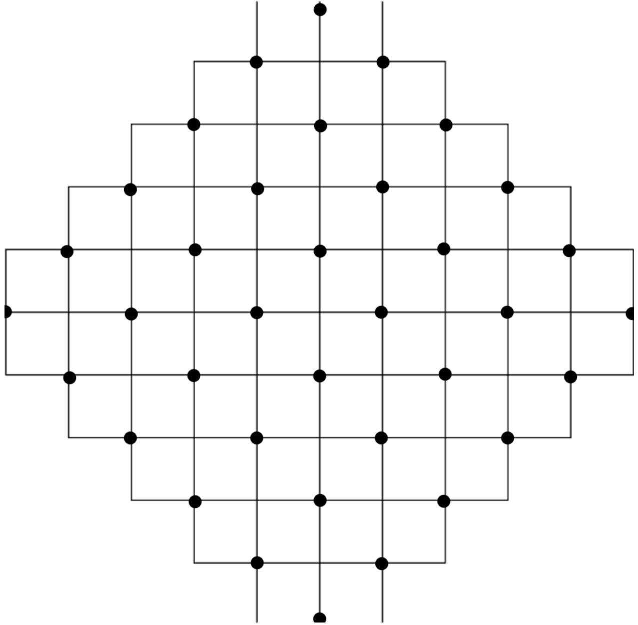


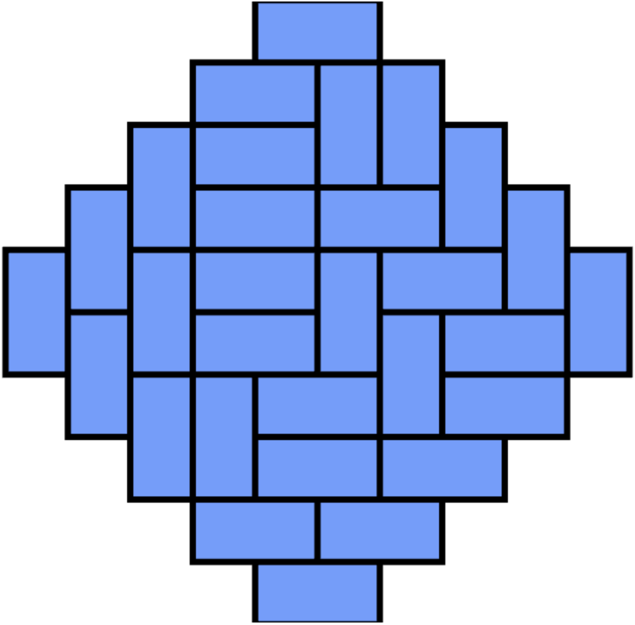


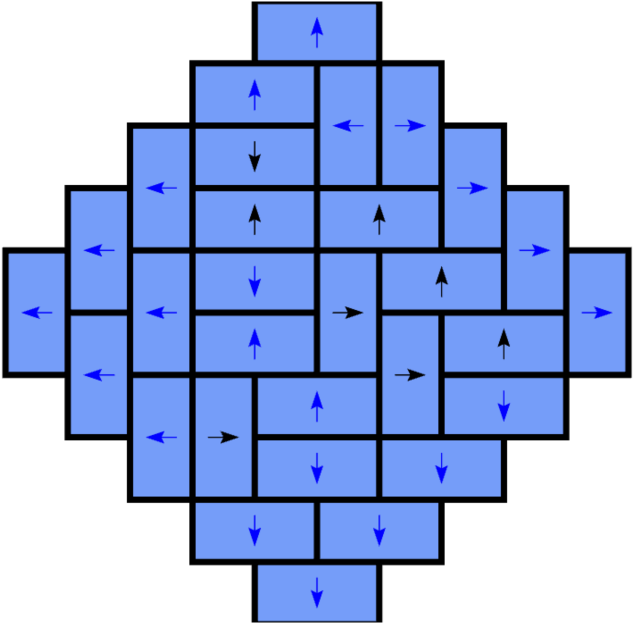


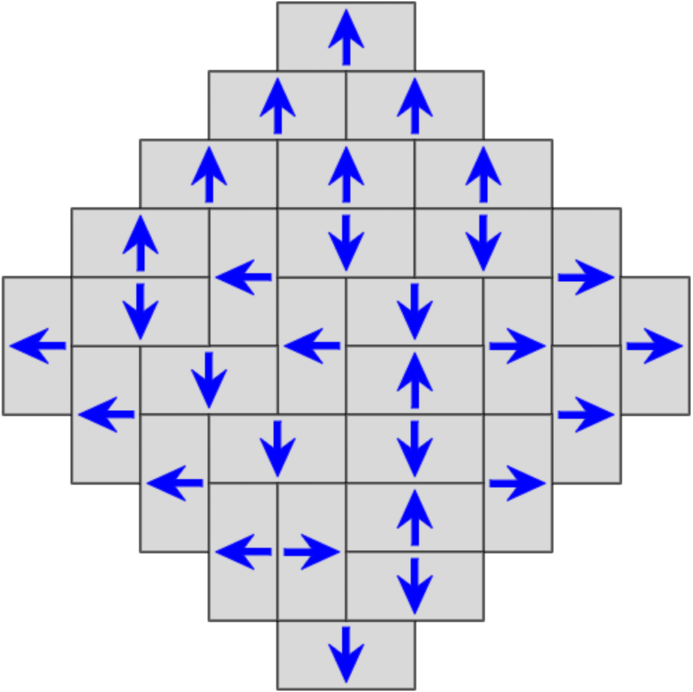


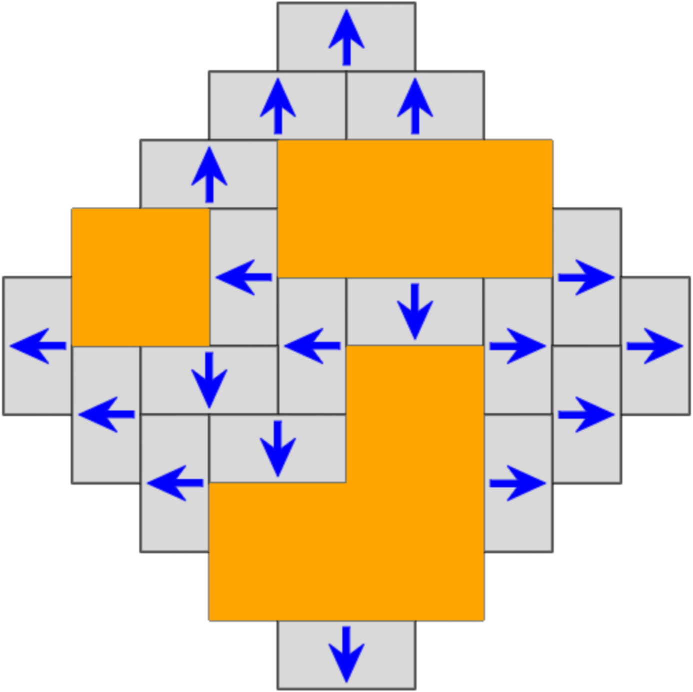


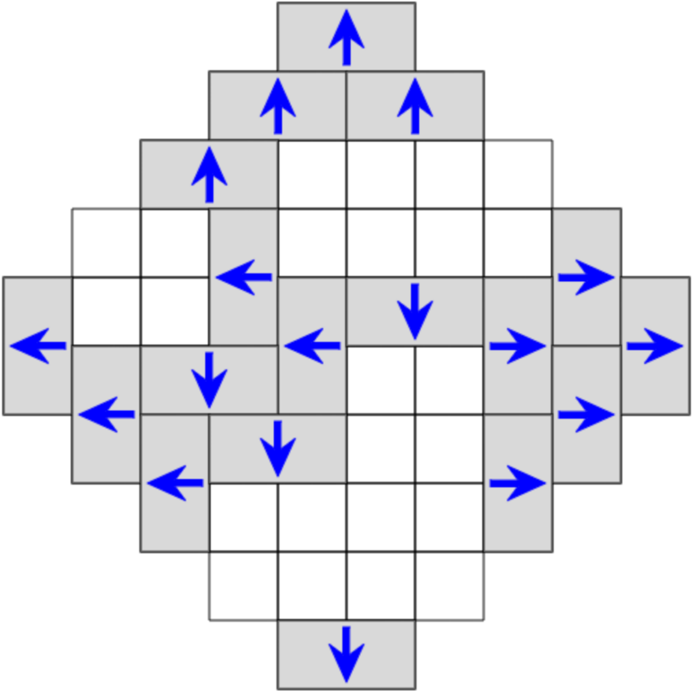


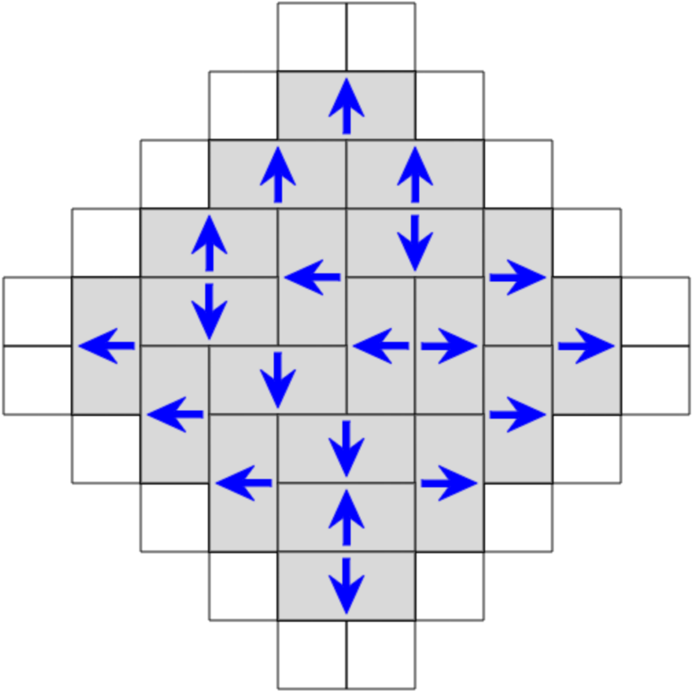


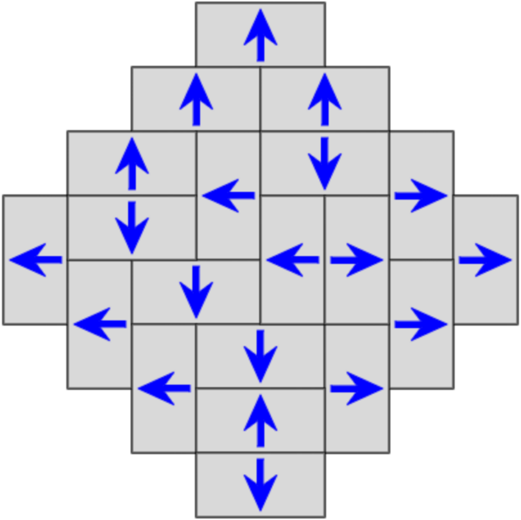


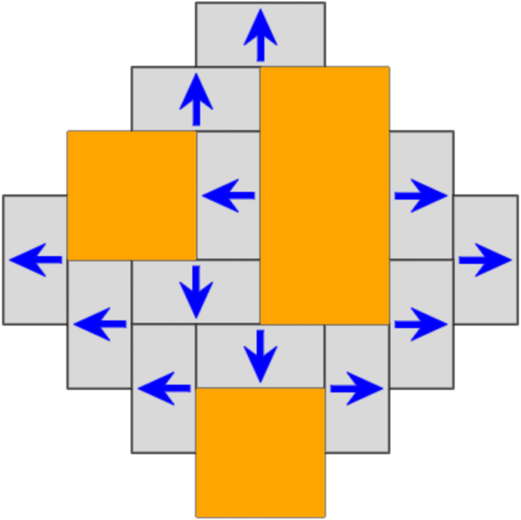


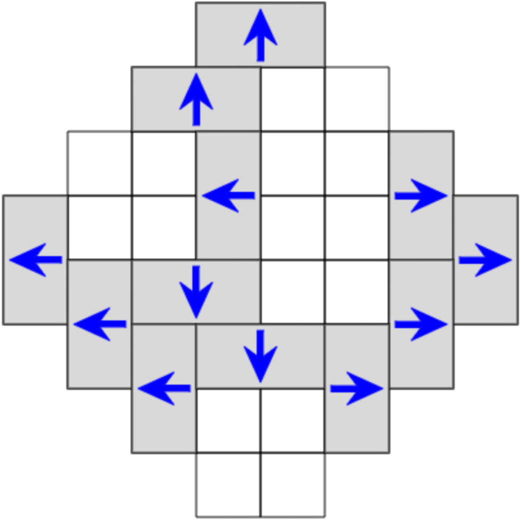


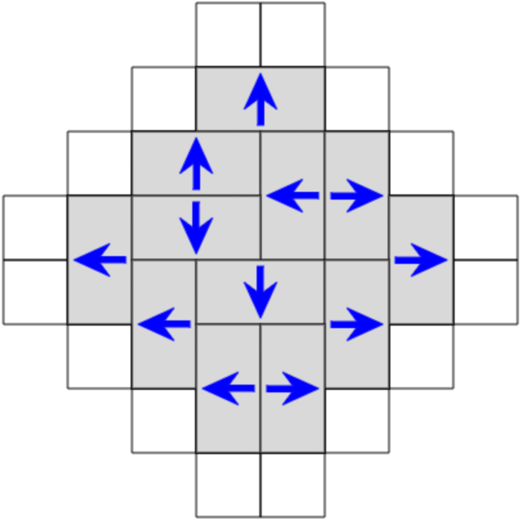


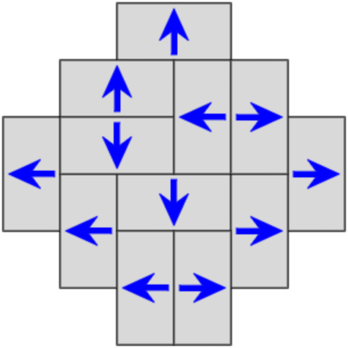


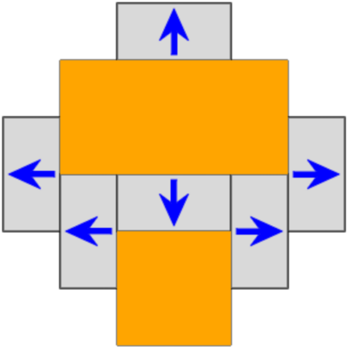


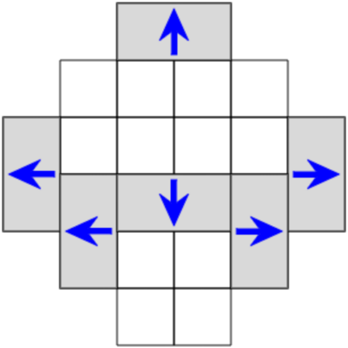


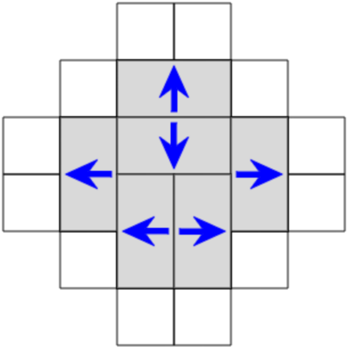


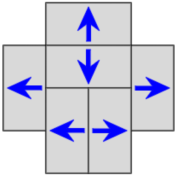


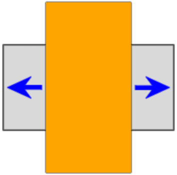


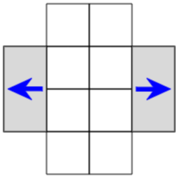


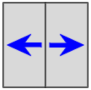


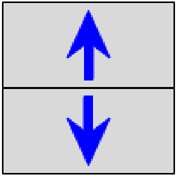
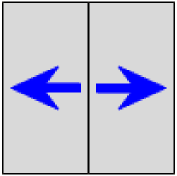


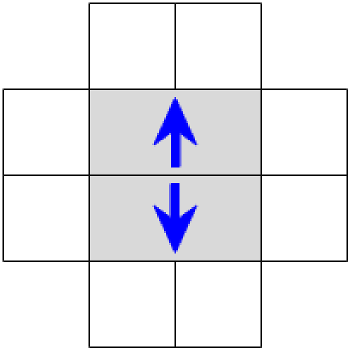
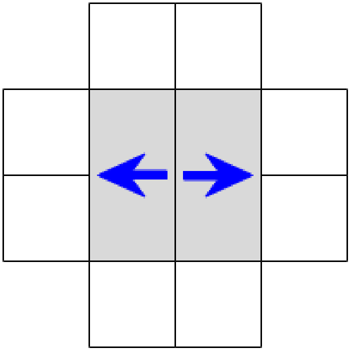


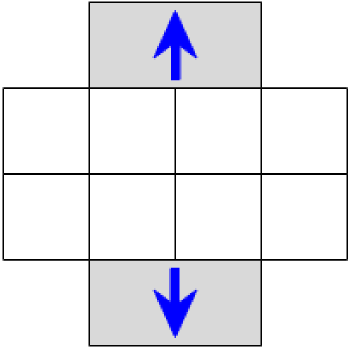


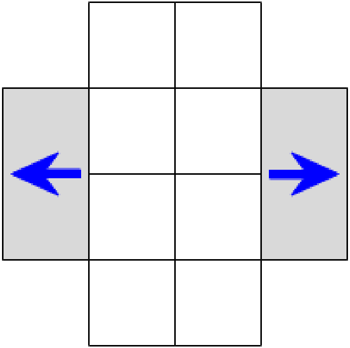


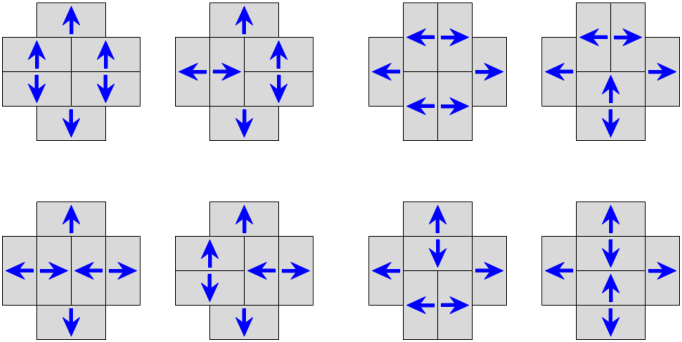


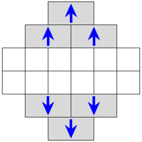
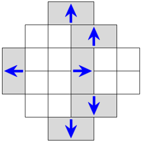
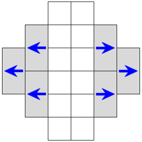
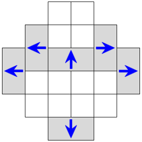
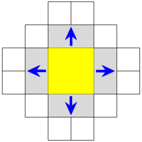
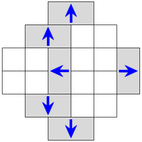
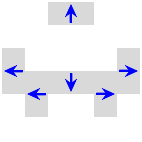
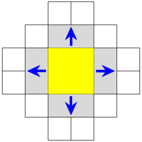



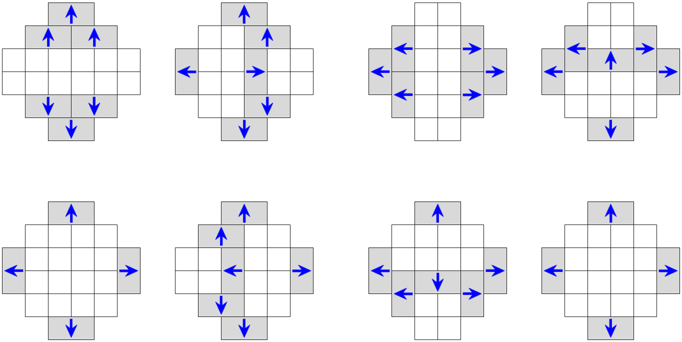



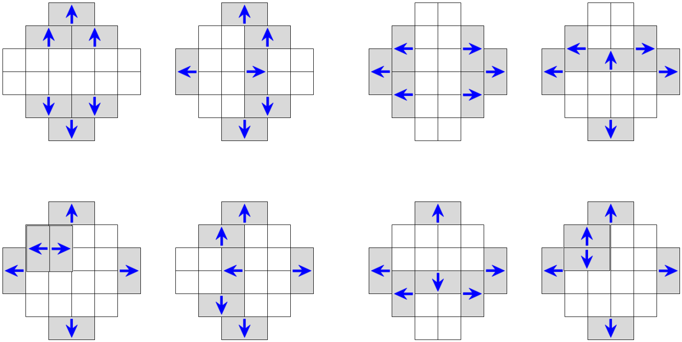






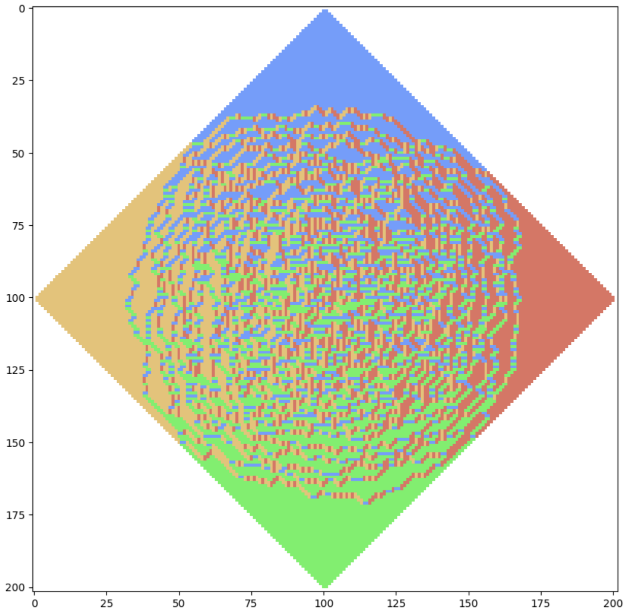


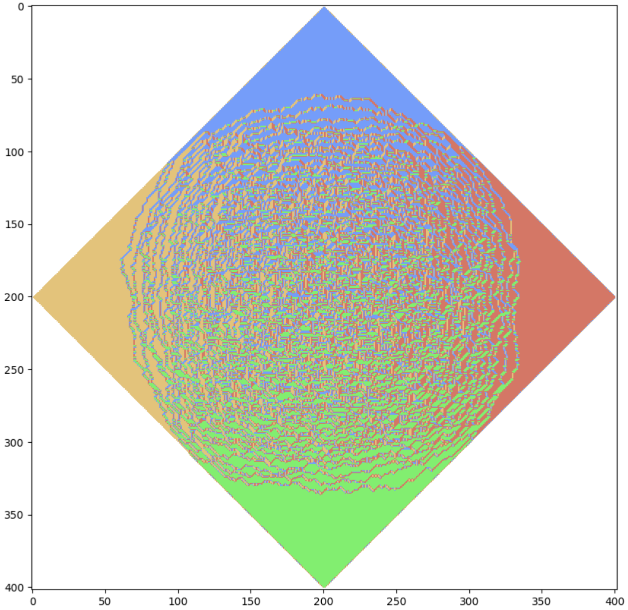


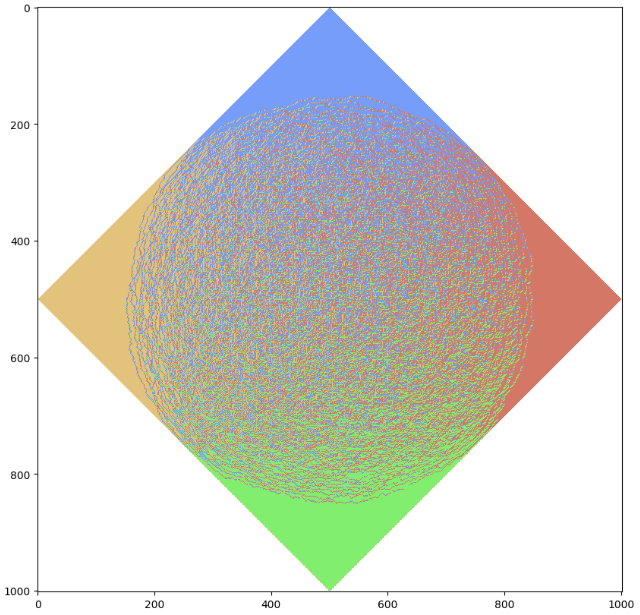
- James Propp
  - Elkies, N., Kuperberg, G., Larsen, M., J. Propp. Alternating-Sign Matrices and Domino Tilings (Part I). Journal of Algebraic Combinatorics 1, 111–132 (1992)
  - Jockusch, W., Propp, J., & Shor, P. (1998). Random domino tilings and the arctic circle theorem. arXiv preprint math/9801068.

- Burkard Polster
  - Mathologer, The ARCTIC CIRCLE THEOREM or Why do physicists play dominoes?, https://www.youtube.com/watch?v=Yy7Q8IWNf

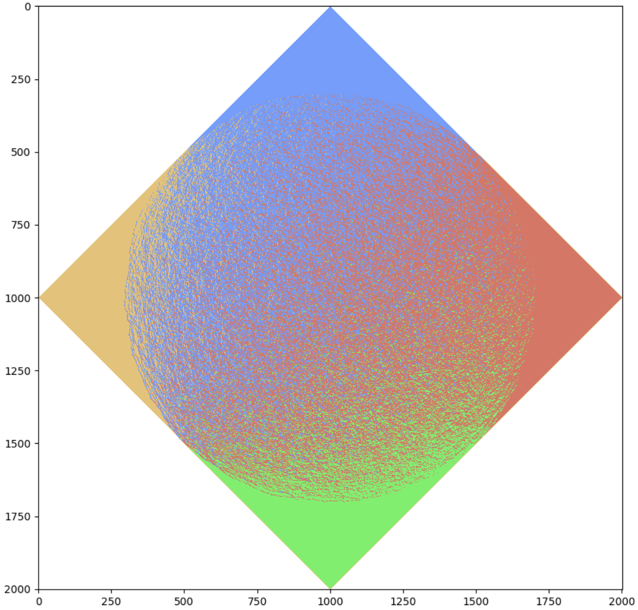
- Dmytro Fedoriaka,
  - https://fedimser.github.io/adt/adt.html


0
25
50
75
100
125
150
175
200
0
25
50
75
100
125
150
175
200


0
50
100
150
200
250
300
350
400
0
50
100
150
200
250
300
350
400



0
200
400
600
800
1000
0
200
400
600
800
1000


0
250
500
750
1000
1250
1500
1750
2000
0
250
500
750
1000
1250
1500
1750
2000